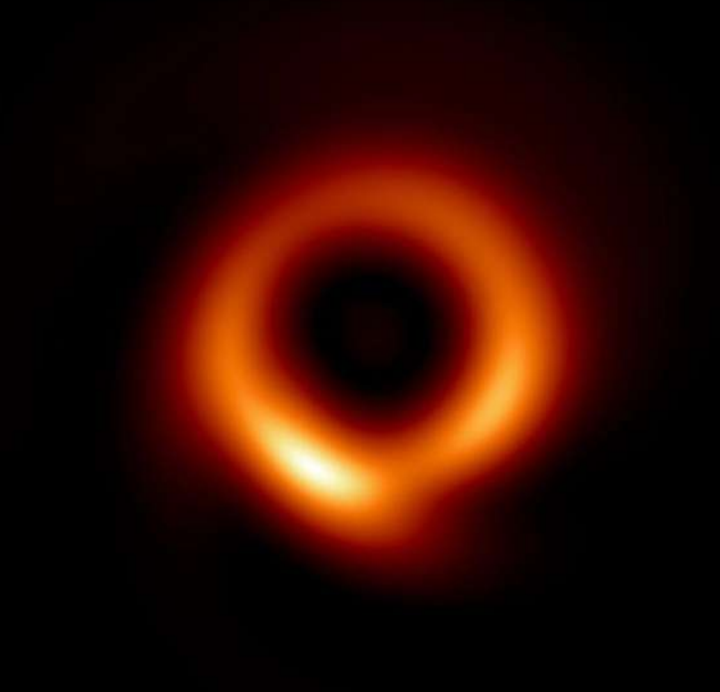
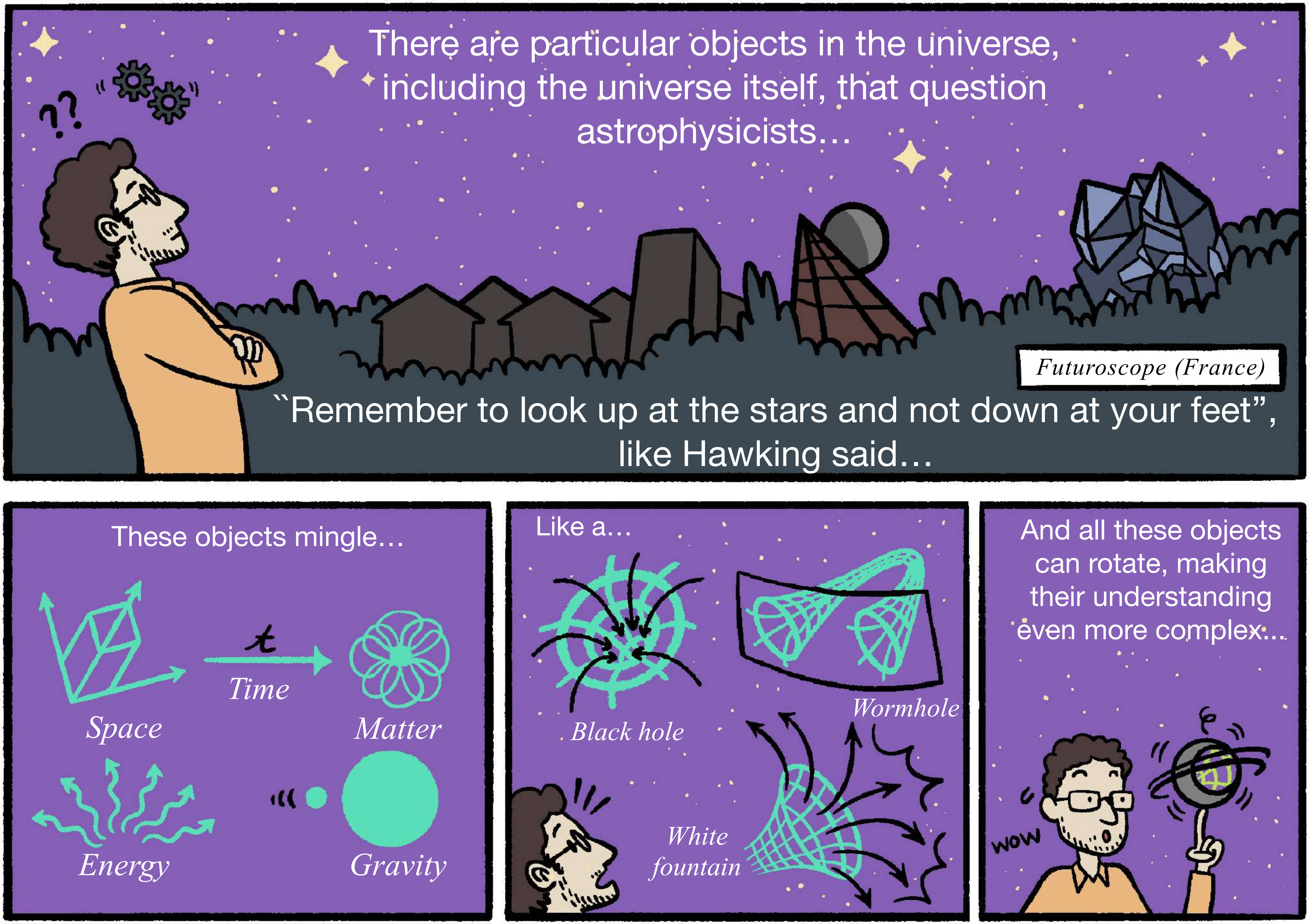
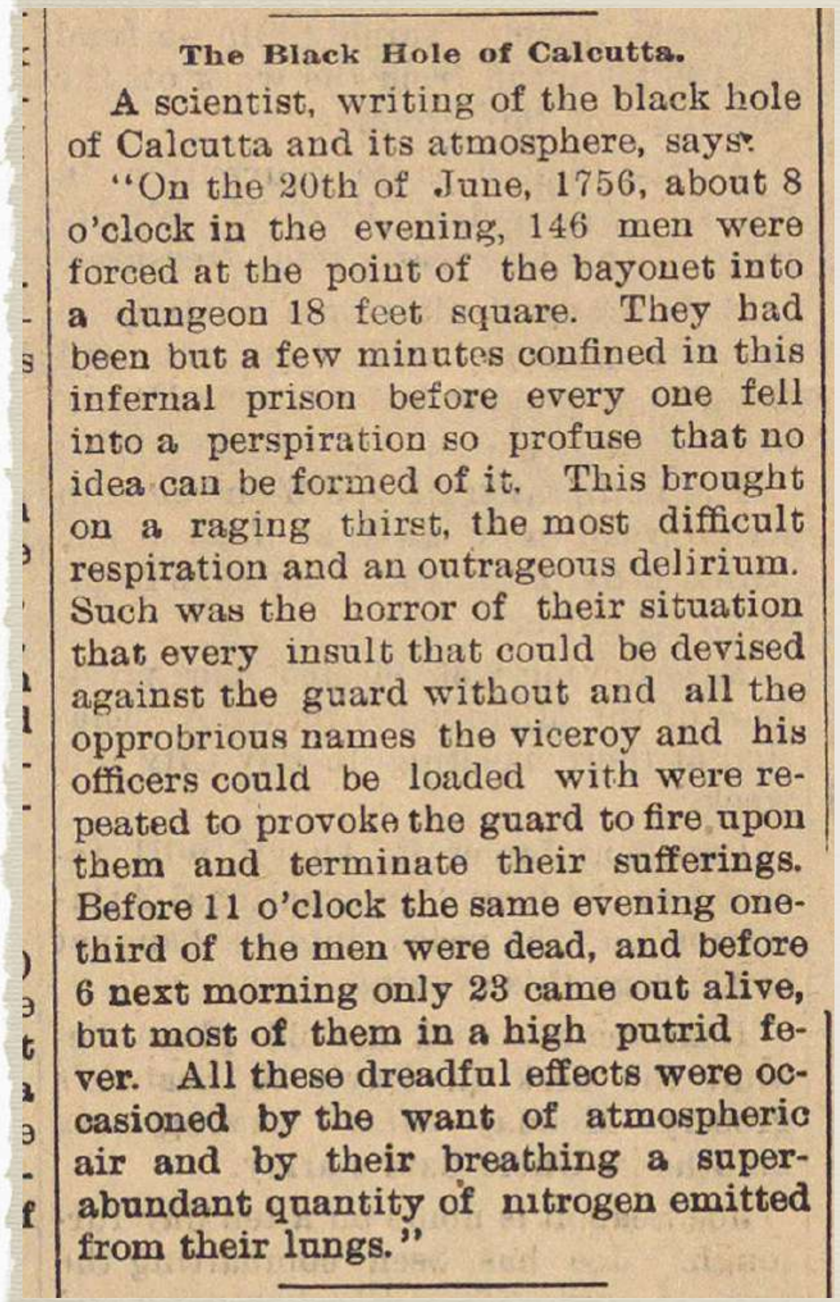
A long time ago in a galaxy far,
far away....




There are particular objects in the universe, including the universe itself, that question astrophysicists…
Futuroscope (France)
`Remember to look up at the stars and not down at your feet", like Hawking said…
These objects mingle…
t
Space
Time
Matter
Energy
Gravity
Like a…
Black hole
Wormhole
White fountain
And all these objects can rotate, making their understanding even more complex…
WOW

The Black Hole Prison of Fort William in Calcutta

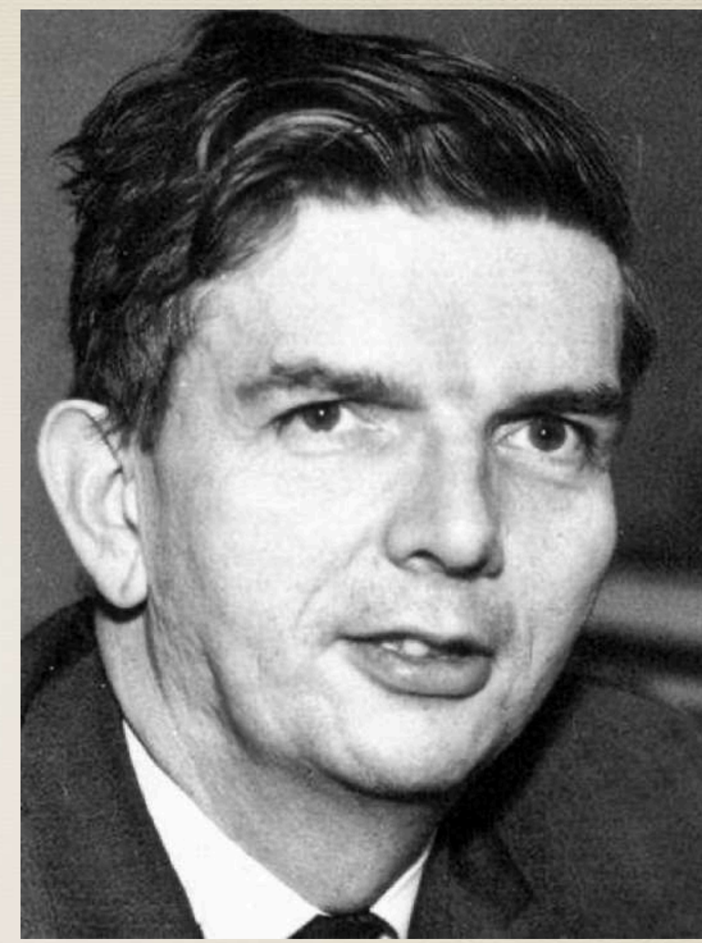
Physicist Robert Dicke in 1961 spoke of peculiar celestial bodies (gravitationally completely collapsed stars) similar to ``the black hole of Calcutta".



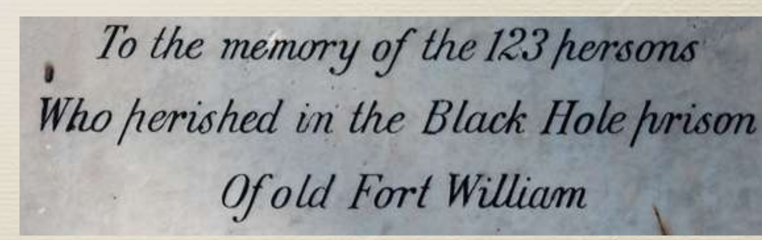
The Black Hole of Calcutta.

A scientist, writing of the black hole of Calcutta and its atmosphere, says:

"On the 20th of June, 1756, about 8 o'clock in the evening, 146 men were forced at the point of the bayonet into a dungeon 18 feet square. They had been but a few minutes confined in this infernal prison before every one fell into a perspiration so profuse that no idea can be formed of it. This brought on a raging thirst, the most difficult respiration and an outrageous delirium. Such was the horror of their situation that every insult that could be devised against the guard without and all the opprobrious names the viceroy and his officers could be loaded with were repeated to provoke the guard to fire upon them and terminate their sufferings. Before 11 o'clock the same evening one-third of the men were dead, and before 6 next morning only 23 came out alive, but most of them in a high putrid fever. All these dreadful effects were occasioned by the want of atmospheric air and by their breathing a superabundant quantity of nitrogen emitted from their lungs."

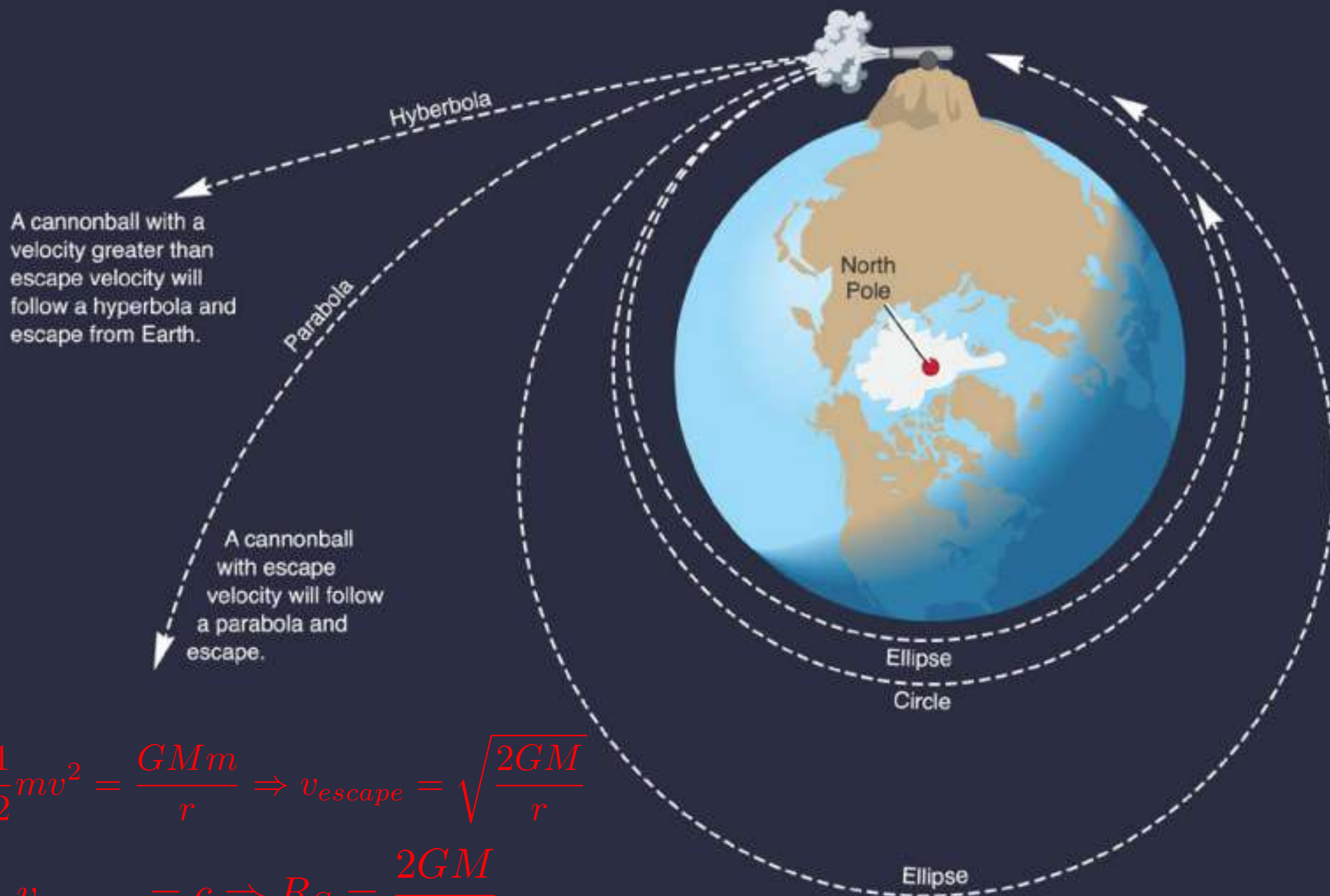


Robert Dicke (1916-1997).



``The Black Hole was a small dungeon in the old Fort William in Calcutta where troops of the Nawab of Bengal, Siraj ud-Daulah, held British prisoners of war after the capture of the fort on 20 June 1756. British and Anglo-Indian soldiers and civilians were held overnight in conditions so cramped that many died from suffocation, heat exhaustion and crushing."

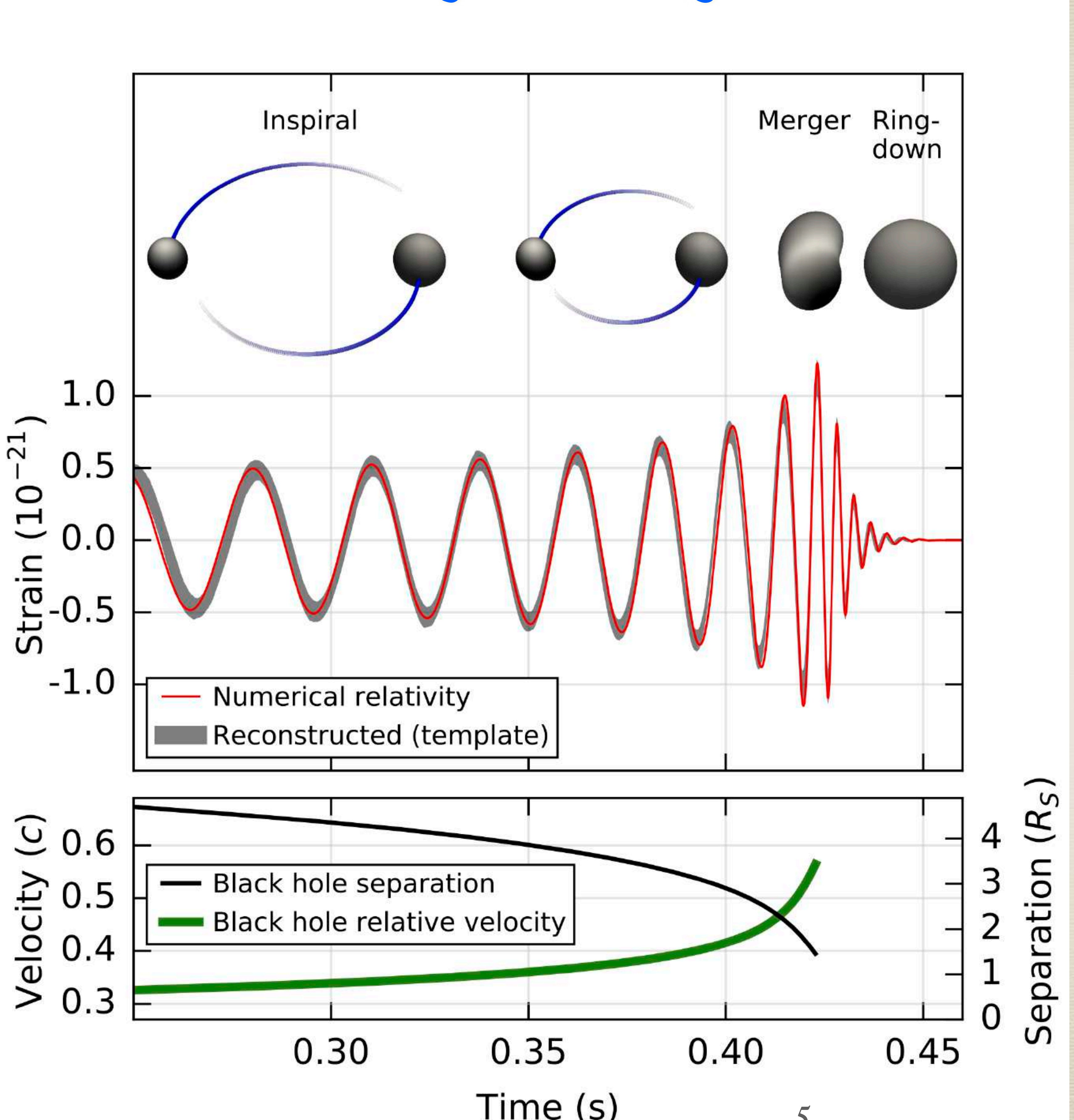
Escape Velocity and Schwarzschild Horizon



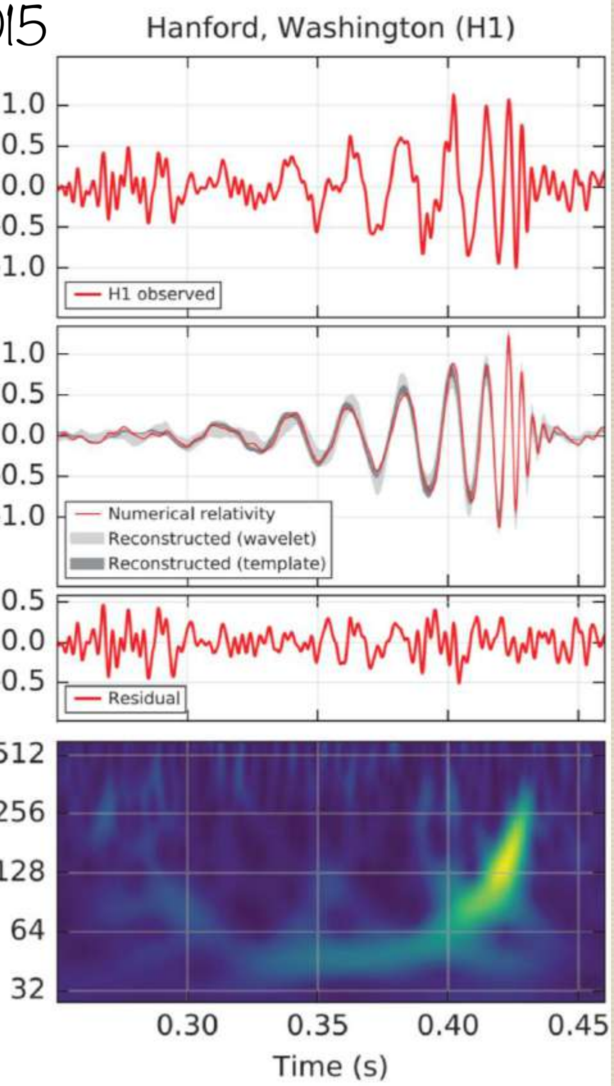
$$\frac{1}{2}mv^2 = \frac{GMm}{r} \Rightarrow v_{escape} = \sqrt{\frac{2GM}{r}}$$

$$v_{escape} = c \Rightarrow R_S = \frac{2GM}{c^2}$$

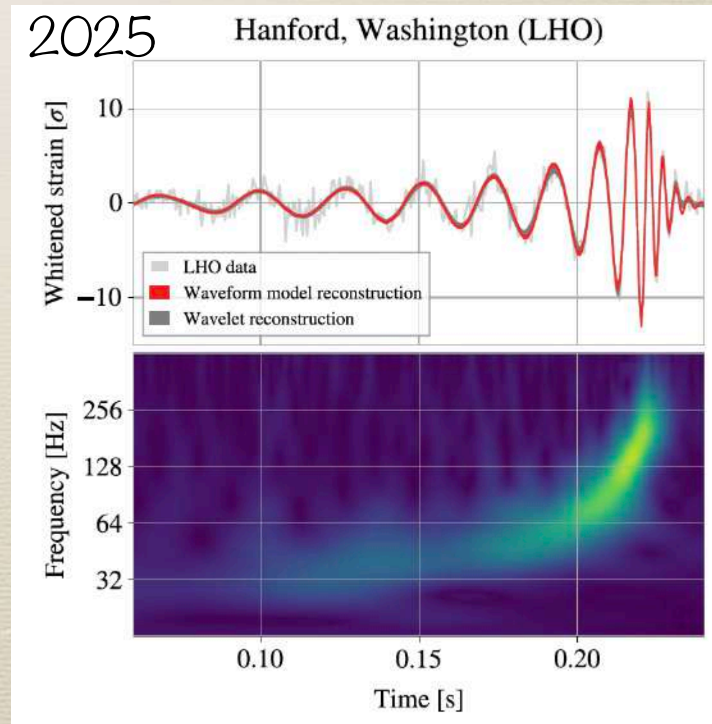
A Black Holes Merger as seen by the LIGO Interferometer in 2015/2025



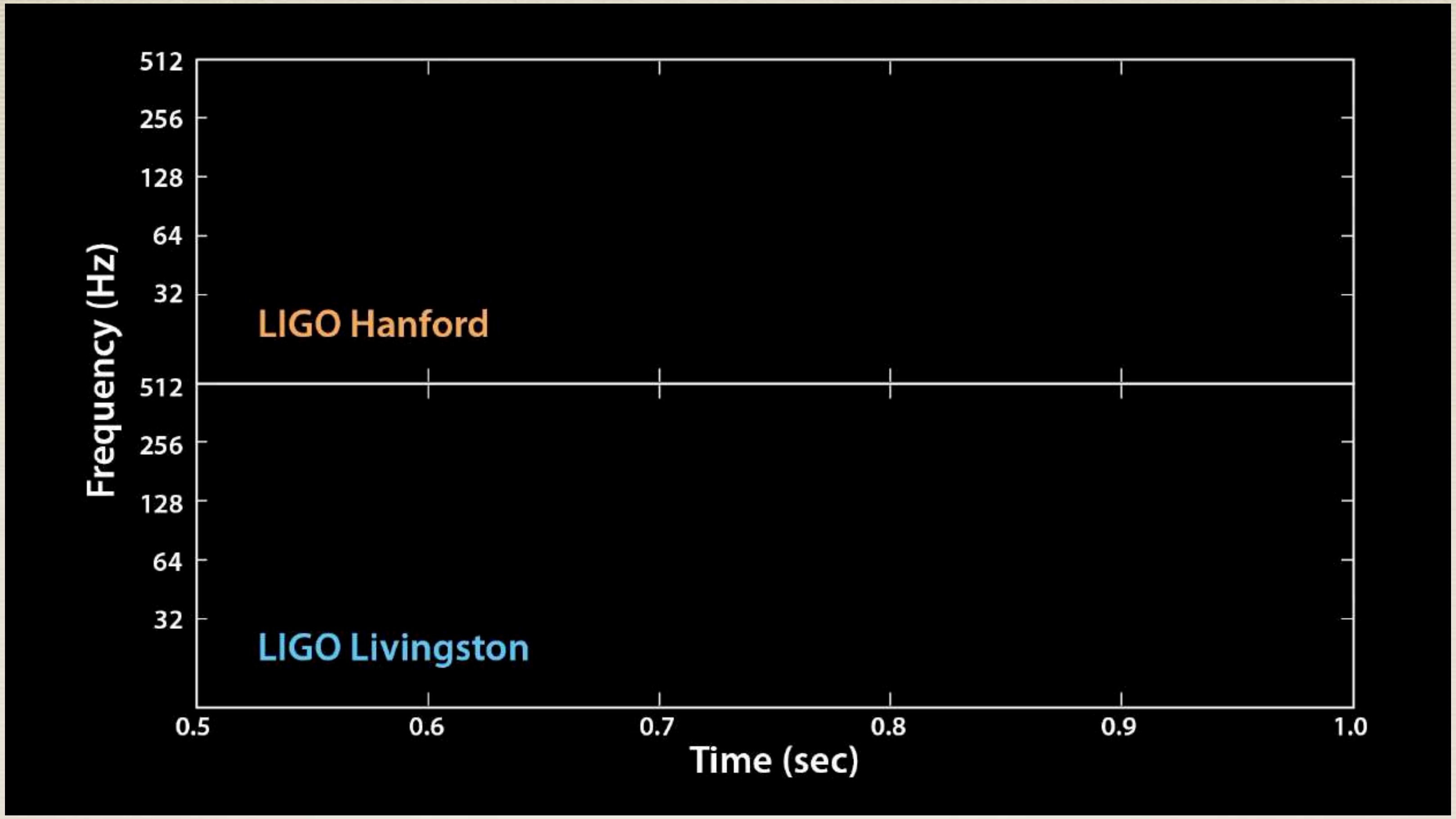
2015



2025



The Sound of Two Black Holes Colliding

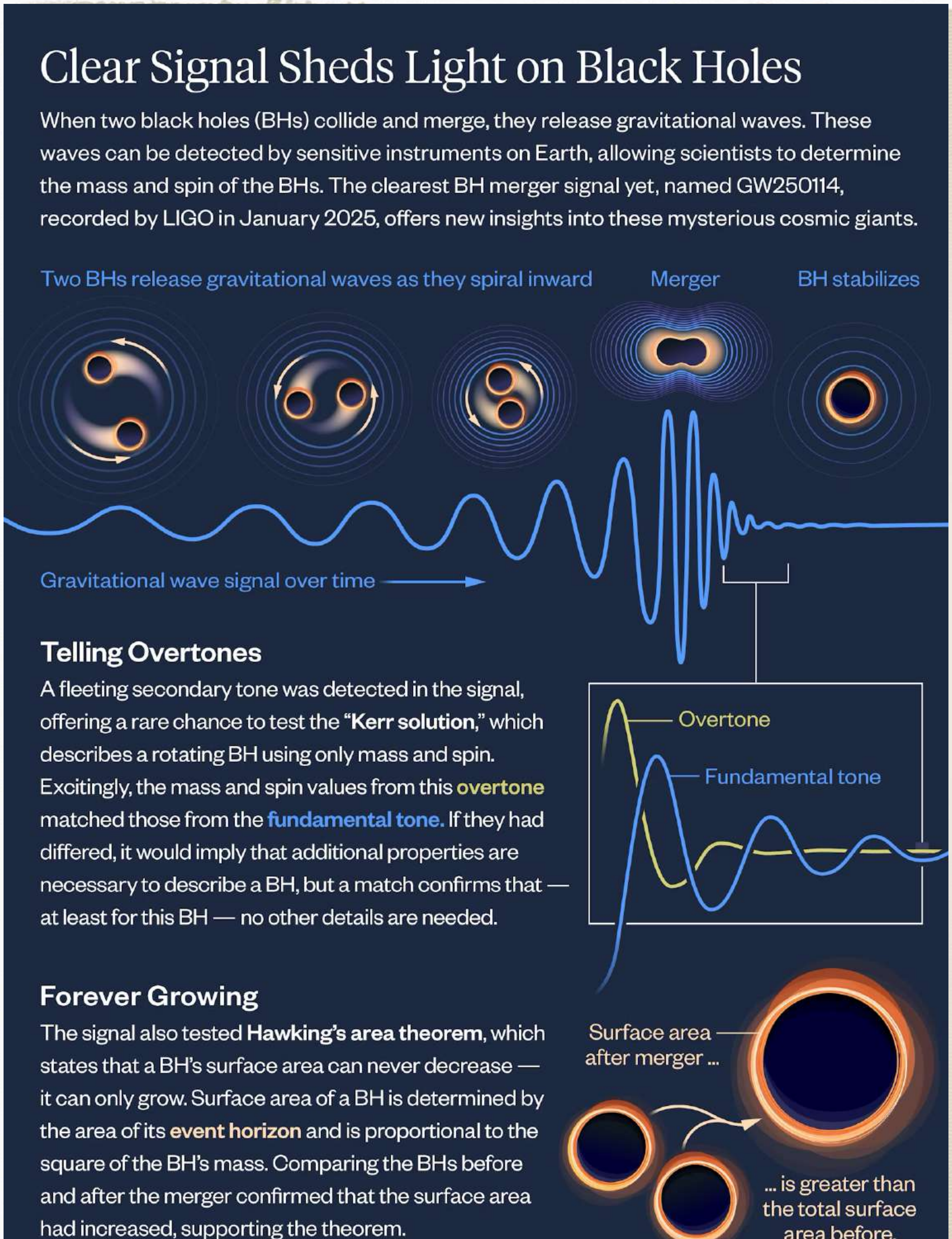


Audio Credit: Caltech/MIT/LIGO Lab

Gravitational waves sent out from a pair of colliding black holes have been converted to sound waves, as heard in this animation. On September 14, 2015, LIGO observed gravitational waves from the merger of two black holes, each about 30 times the mass of our sun. The incredibly powerful event, which released 50 times more energy than all the stars in the observable universe, lasted only fractions of a second.

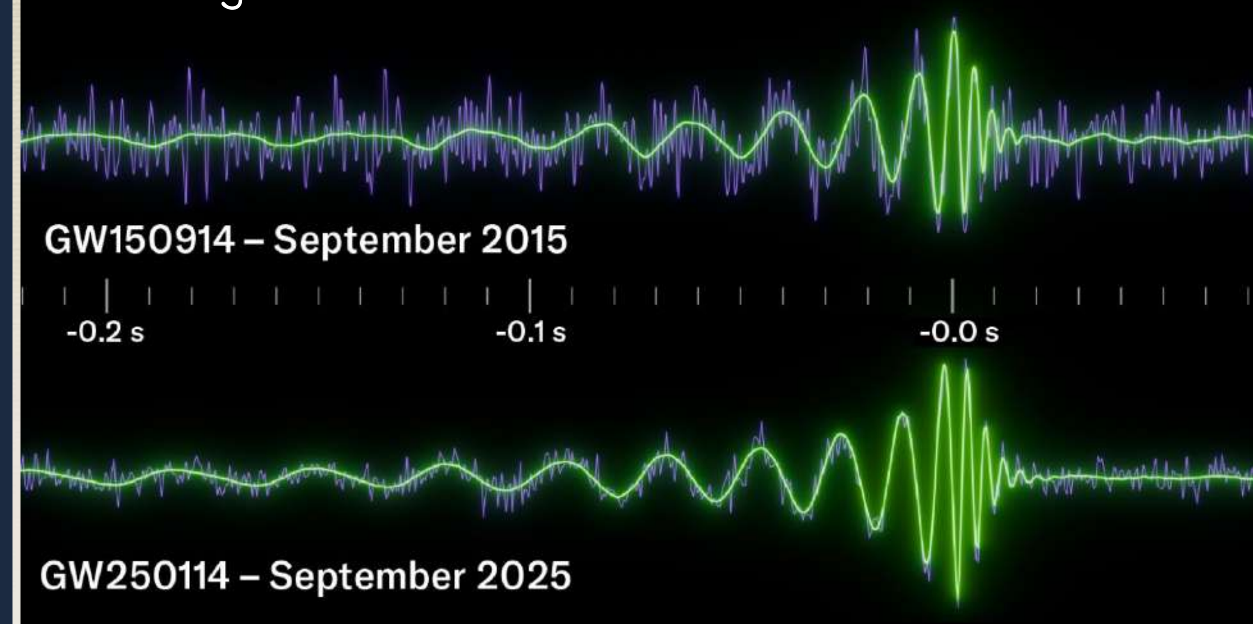
In the first two runs of the animation, the sound-wave frequencies exactly match the frequencies of the gravitational waves. The second two runs of the animation play the sounds again at higher frequencies that better fit the human hearing range. The animation ends by playing the original frequencies again twice.

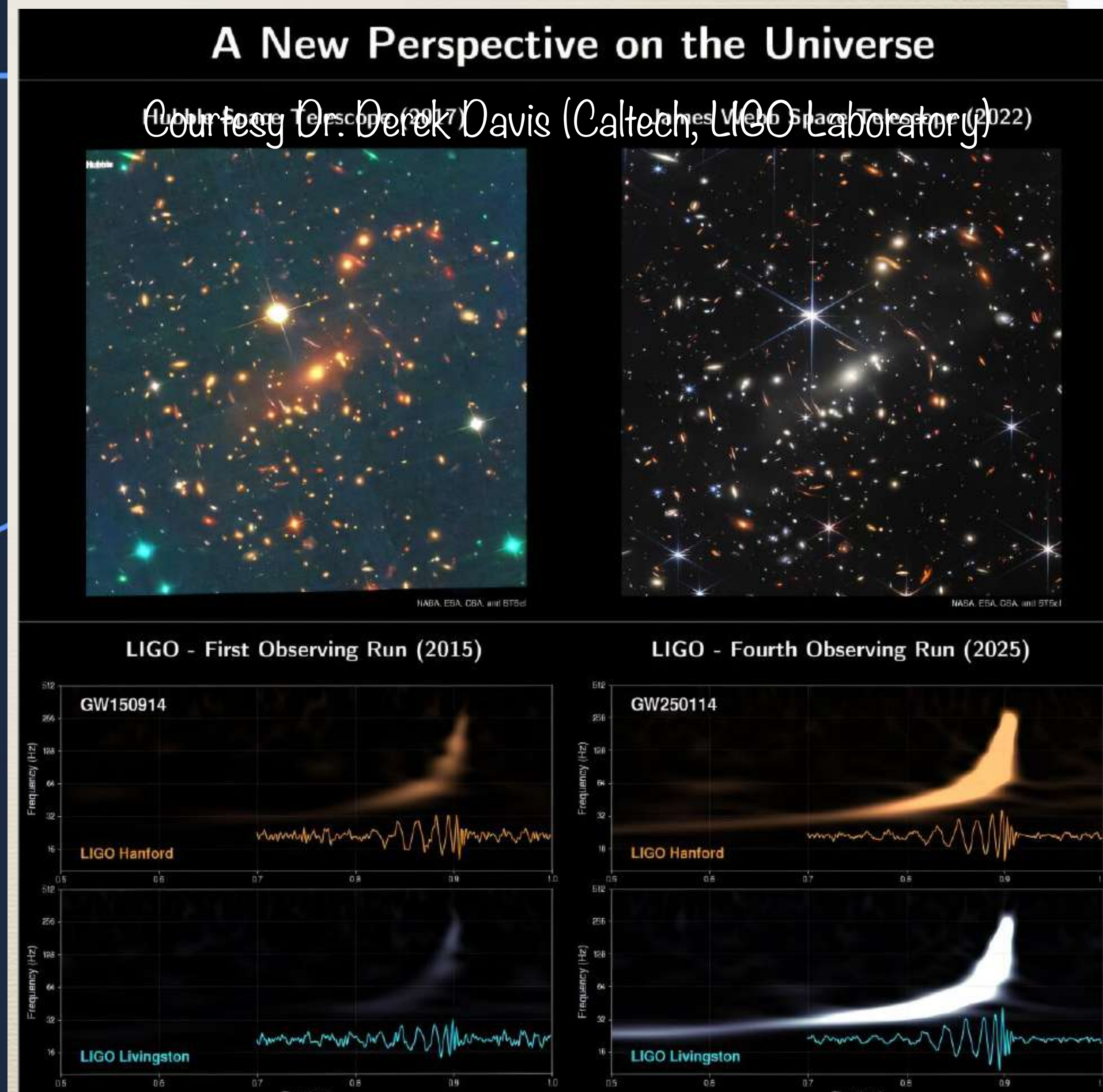
As the black holes spiral closer and closer in together, the frequency of the gravitational waves increases. Scientists call these sounds "chirps," because some events that generate gravitation waves would sound like a bird's chirp.



Courtesy L. Reading-Ikkanda/Simons Foundatio

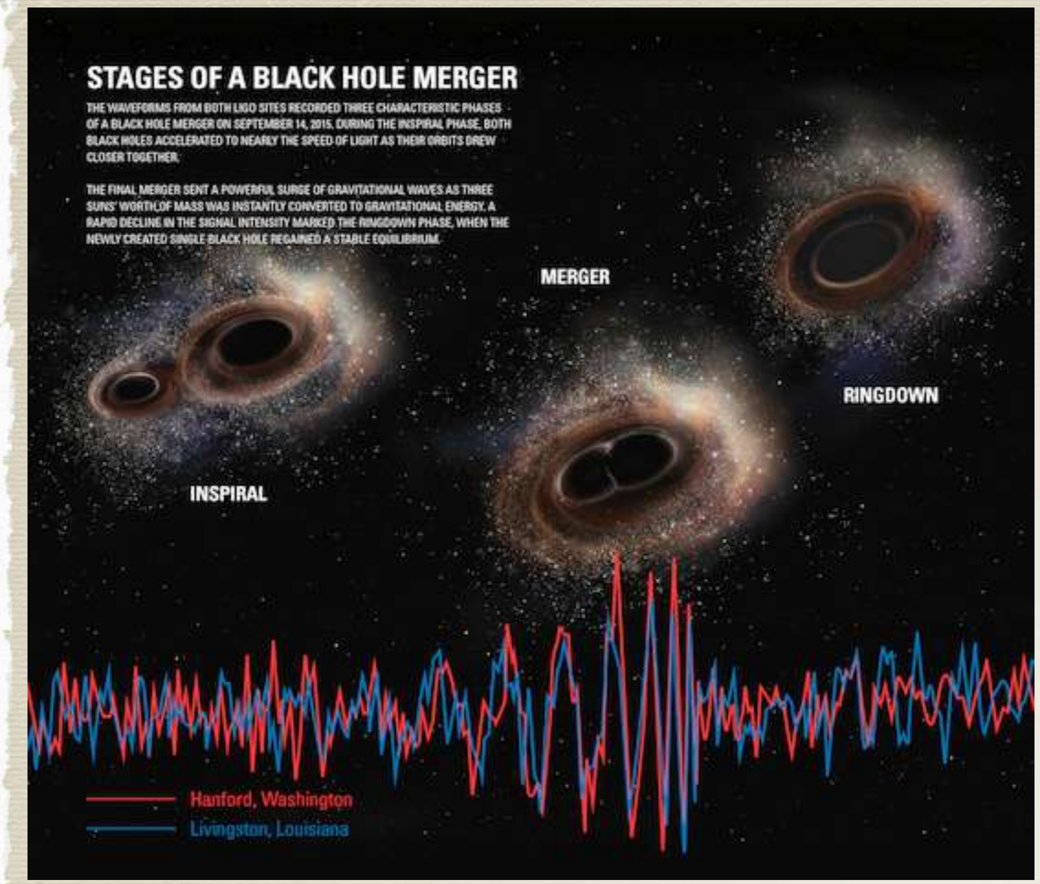
Courtesy LIGO/J. Tissino (GSSI)/R. Hurt (Caltech-IPAC)

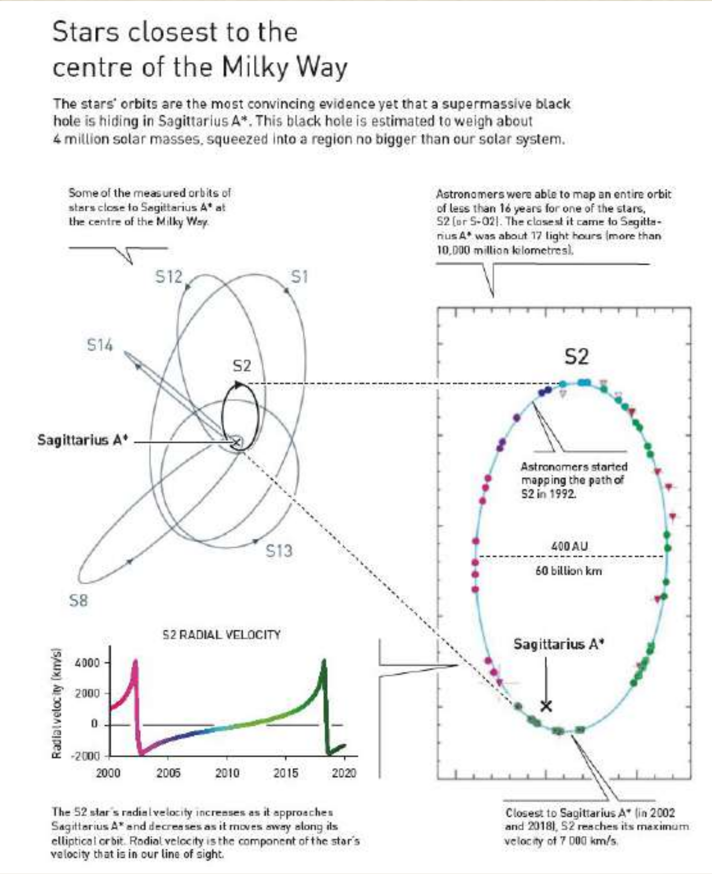


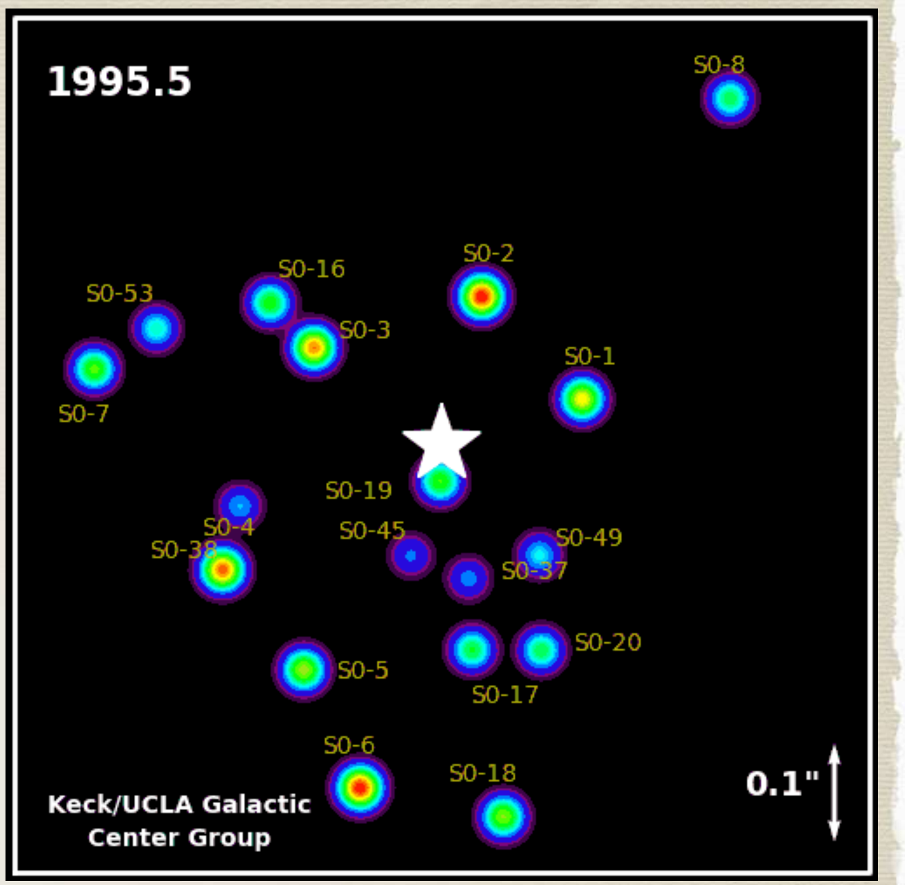


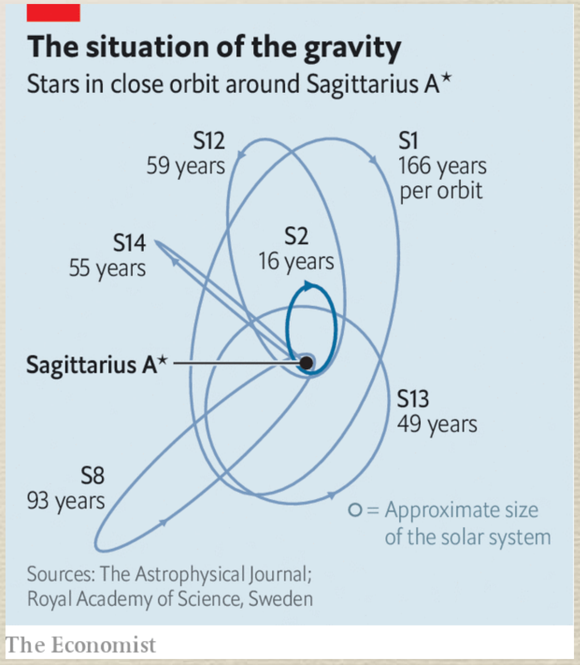
Courtesy Dr. Derek Davis (Caltech, LIGO Laboratory)

A New Golden Age for Black Hole Physics



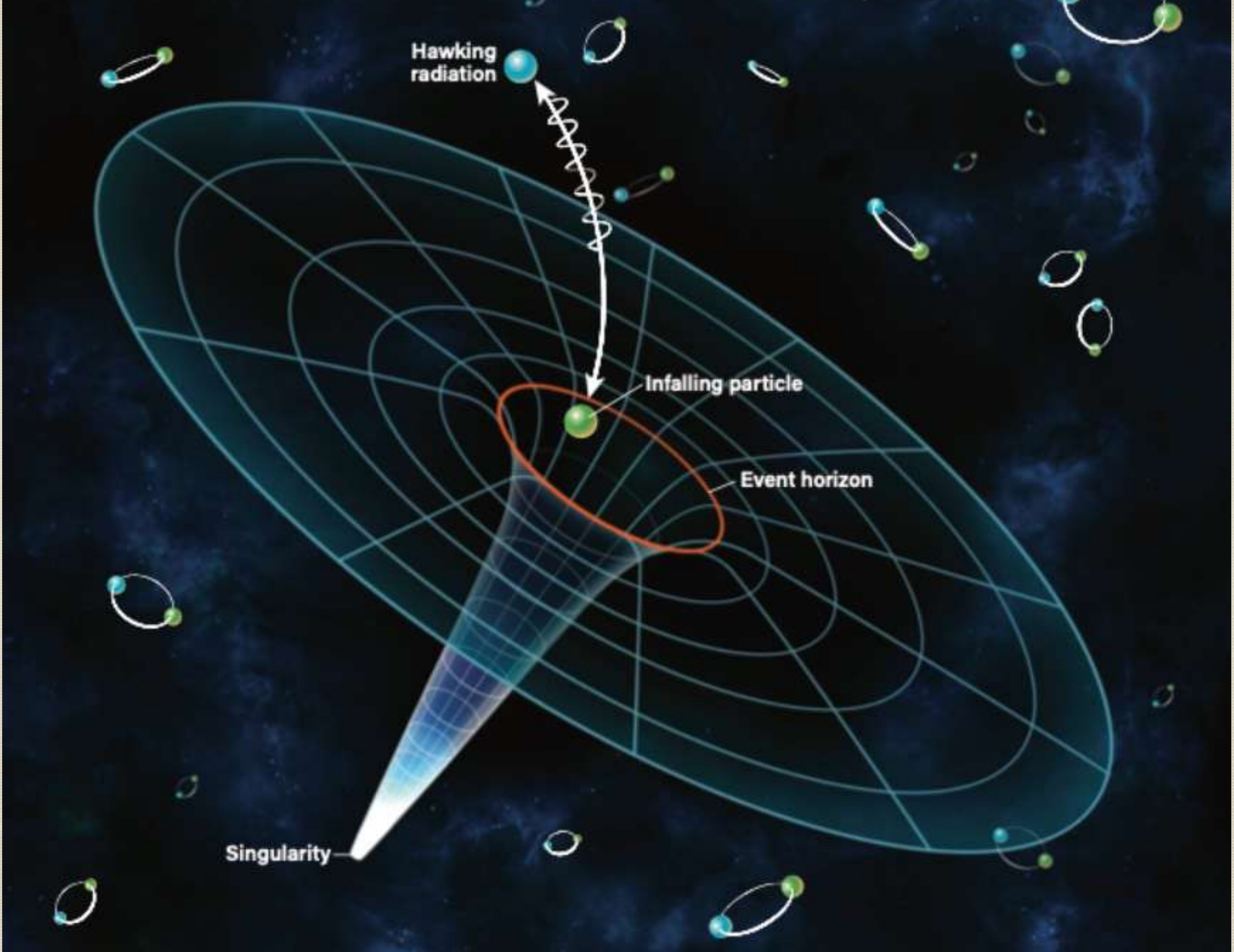






"As you probe ever closer to the horizons of the black holes, Nature might have new surprises in store… » Ulf Danielsson, during the announcement of the Nobel Prize in Physics (October 2020).

Black Holes Should Radiate a Quantum Glow



Credit: Astronomy: Rick Johnson & Roen Kelly

Hawking/Unruh Temperature

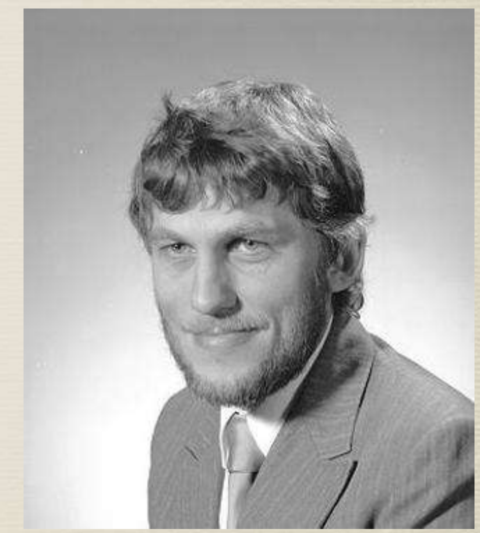
$$E = h\nu = \frac{hc}{\lambda} \qquad \frac{1}{2}mv^2 = \frac{GMm}{r} \Rightarrow v_{escape} = \sqrt{\frac{2GM}{r}} \qquad \lambda \simeq R_S = \frac{2GM}{c^2}$$

$$E \simeq \frac{hc^3}{2GM} \qquad v_{escape} = c \Rightarrow R_S = \frac{2GM}{c^2} \qquad E \simeq k_B T$$

1974

$$T \simeq \frac{hc^3}{2k_B GM} \qquad T_H = \frac{hc^3}{16\pi^2 k_B GM}$$

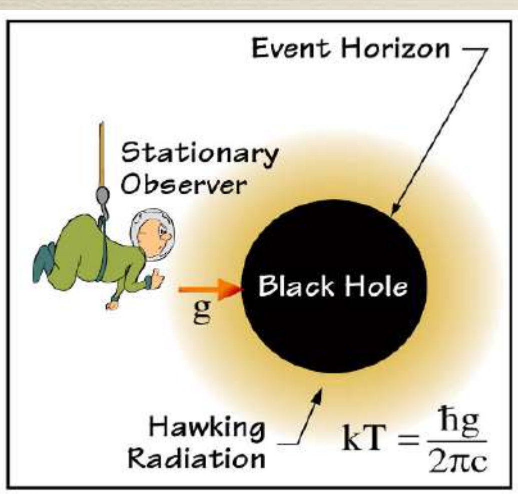
1976



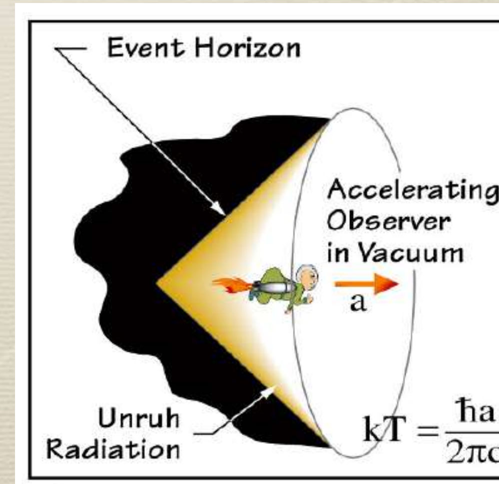
Surface gravity (peeling rate of the rays bundle) = acceleration of a plunging observer at the horizon seen by an asymptotic observer

$$\kappa = \frac{v_{escape}^2(R_S)}{R_S} = \frac{2GM}{R_S} = \frac{c^4}{2GM}$$

$$T_H = \frac{\hbar\kappa}{2\pi k_B c} \qquad T_U = \frac{\hbar a}{2\pi k_B c}$$



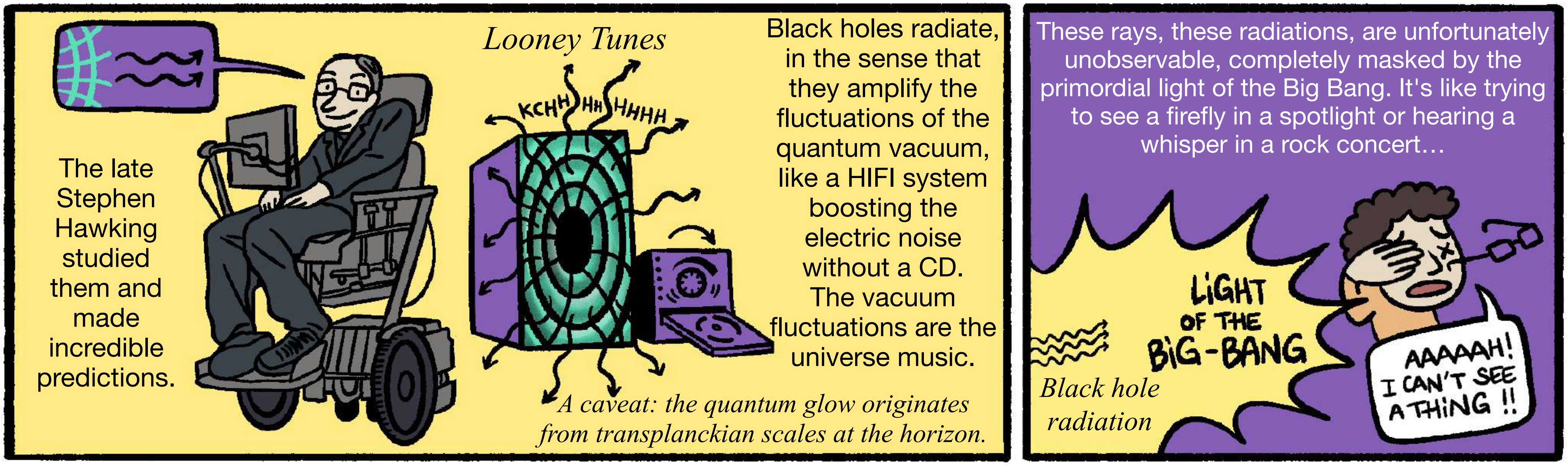
A stationary observer outside the black hole would see the thermal Hawking radiation.



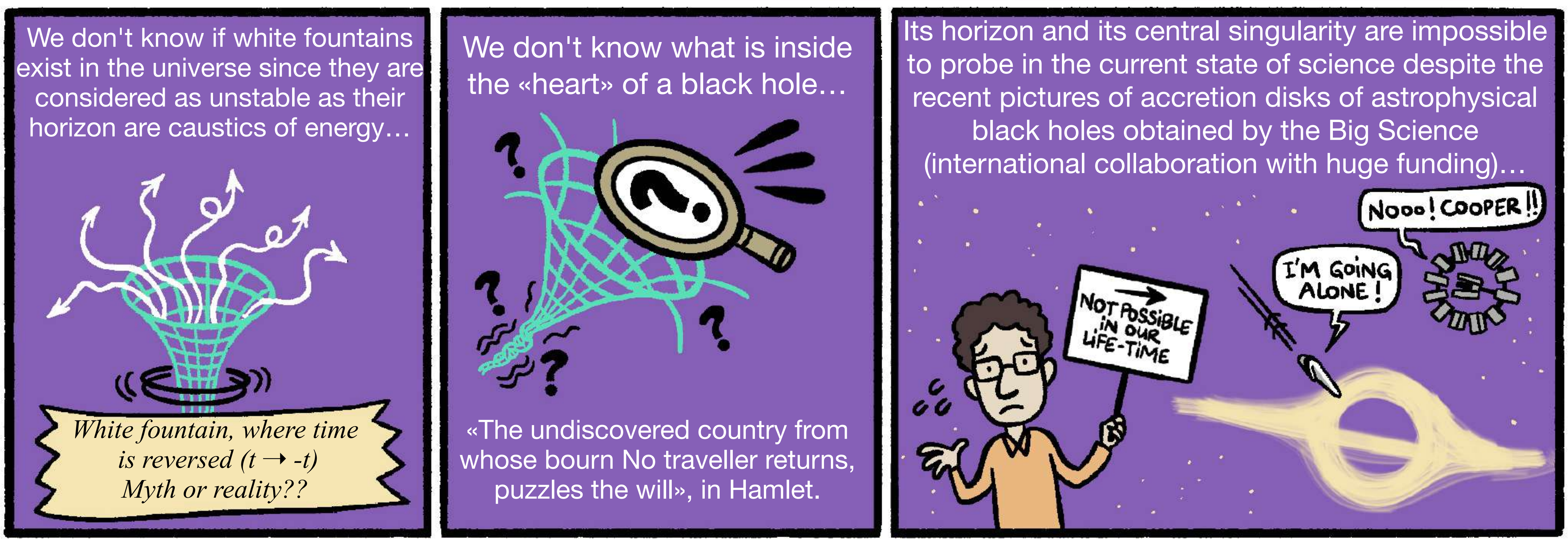
An accelerating observer in vacuum would see a similar Hawking-like radiation called Unruh radiation.

"God not only plays dice, he also sometimes throws the dice where they cannot be seen."

Stephen Hawking



EVERYTHING THAT IS YET UNKNOWN




New Scientist
CAN NEW ALZHEIMER'S TREATMENTS FINALLY CURB DEMENTIA?
WHY THINKING ABOUT YOUR FUTURE SELF CAN CHANGE YOUR LIFE TODAY
WEEKLY 27 May 2023
HOW TO BUILD A BLACK HOLE
Inside the audacious attempts to recreate extreme gravity in the lab
JOIN OUR NEW BOOK CLUB...
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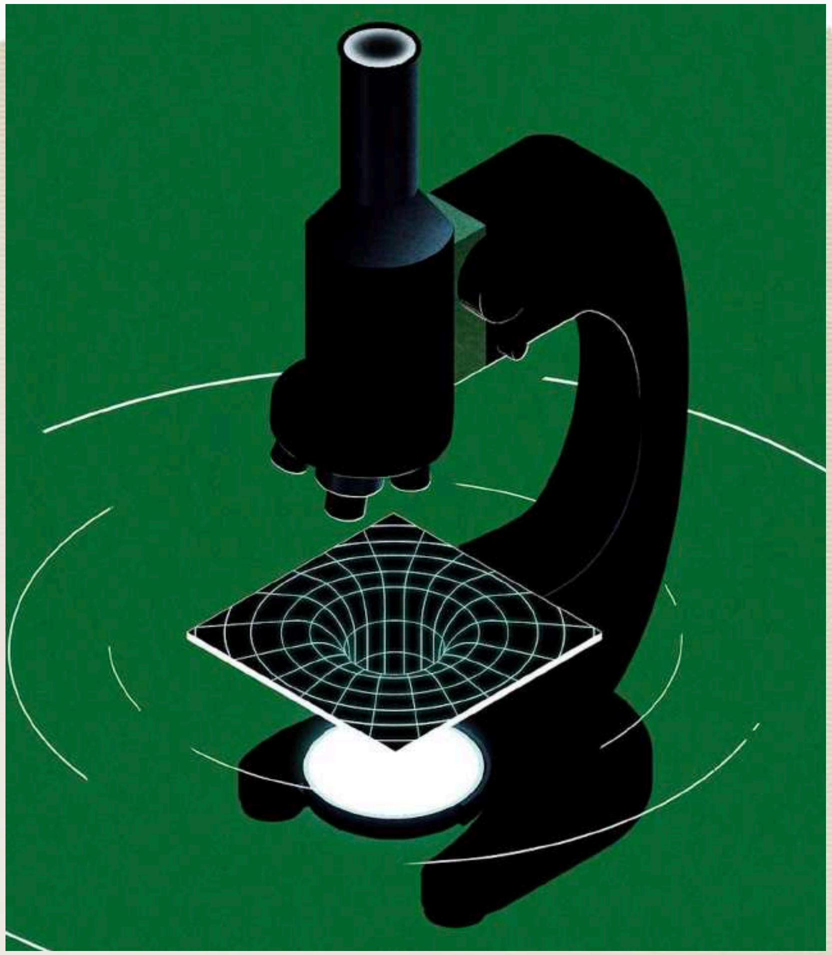
PLUS THE ORIGINS OF KISSING / HOUSE MADE FROM DIAPERS / LOST BEES / DO OCTOPUSES HAVE NIGHTMARES? / BAT CAVES

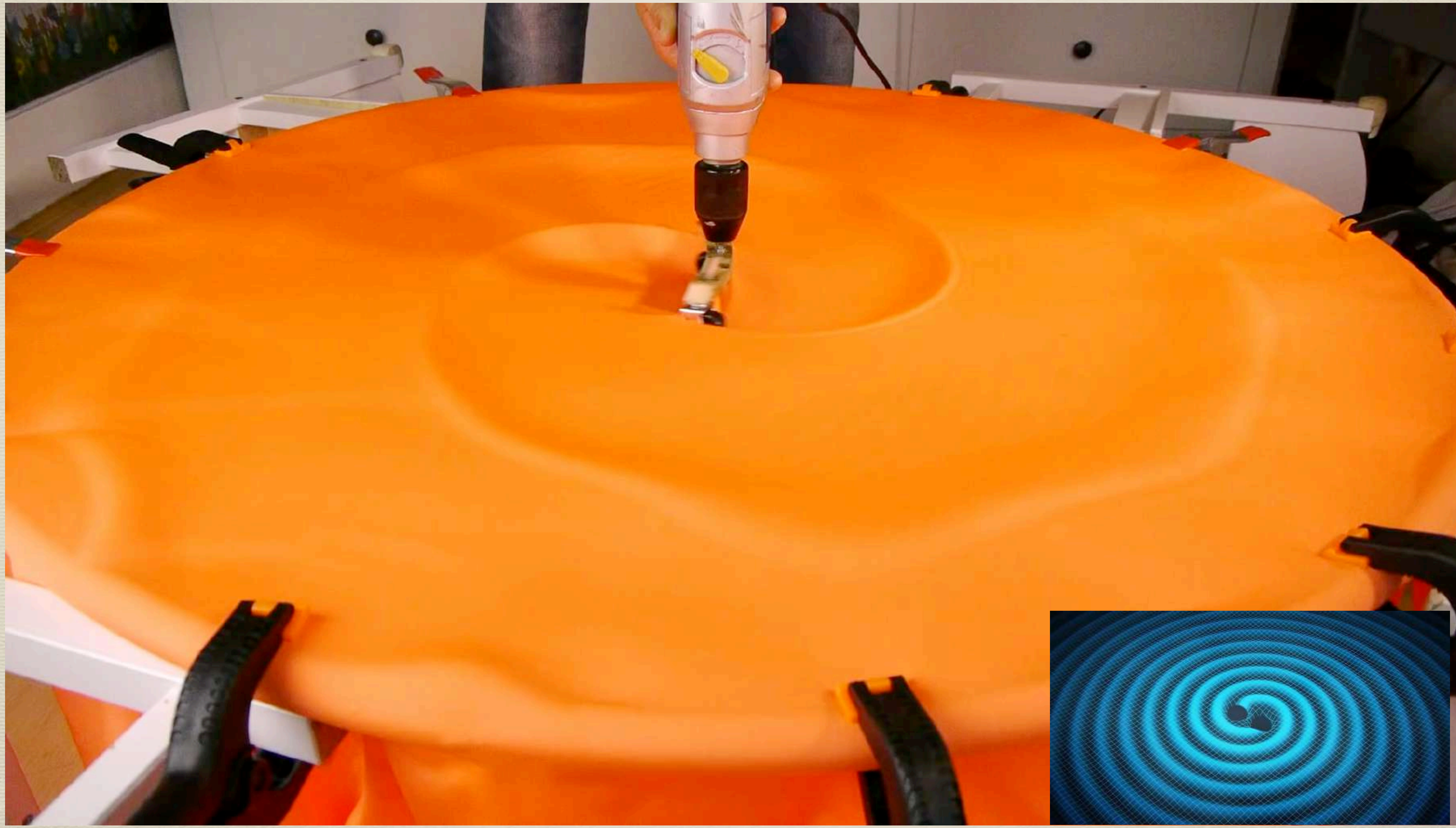


An Economic Black Hole for Tourists

Analogue Simulations at Home

« It seems to me that the test of 'Do we or do we not understand a particular subject in physics?' is 'Can we make a mechanical model of it? »

Lord Kelvin

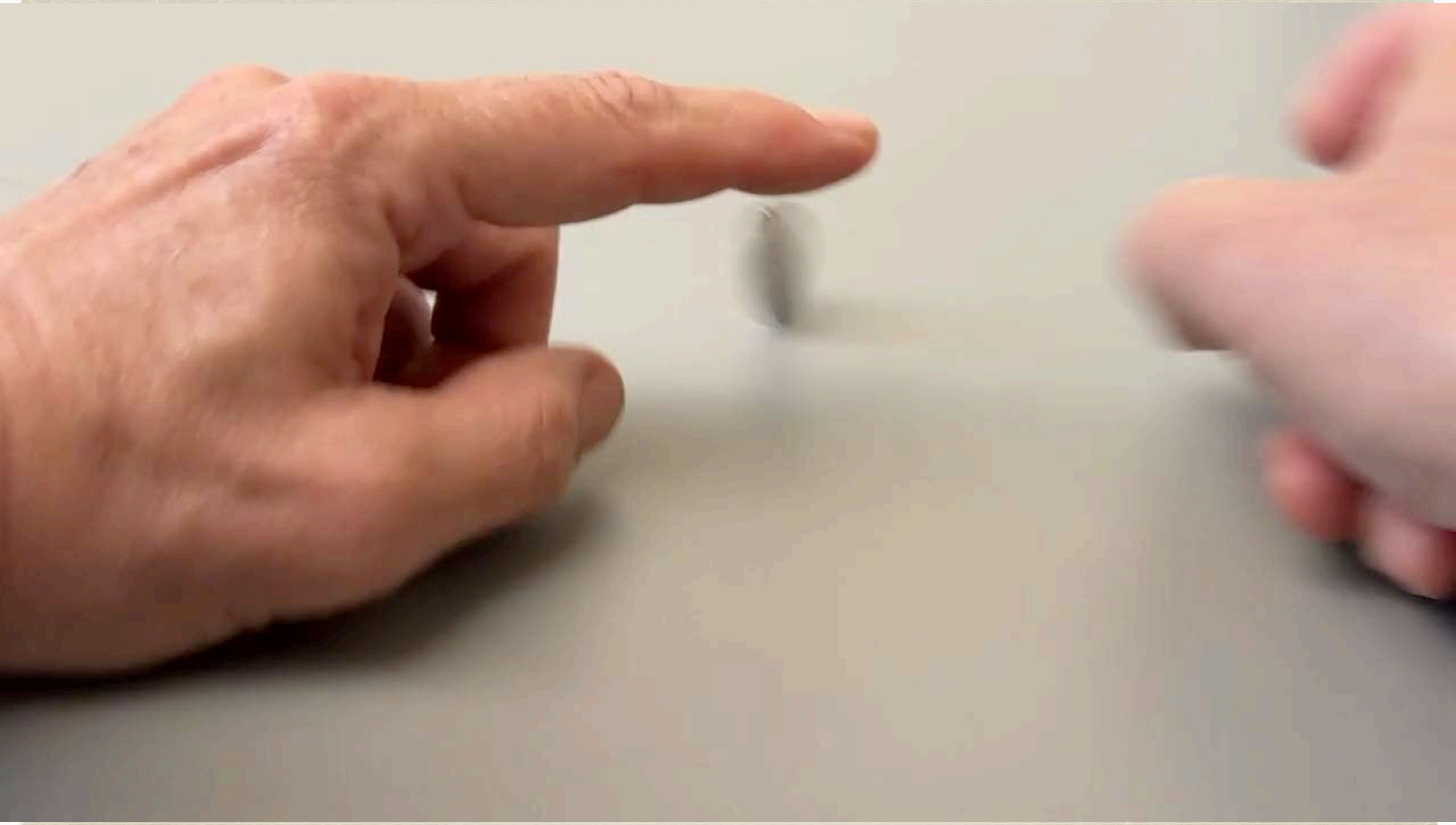


Ingredients : a tablecloth made of stretched spandex lycra + a drill + two roulettes

https://www.youtube.com/watch?v=dw7U3BYMs4U

http://thekidshouldseethis.com/post/69096529285

The Inspiring Power of Analogies



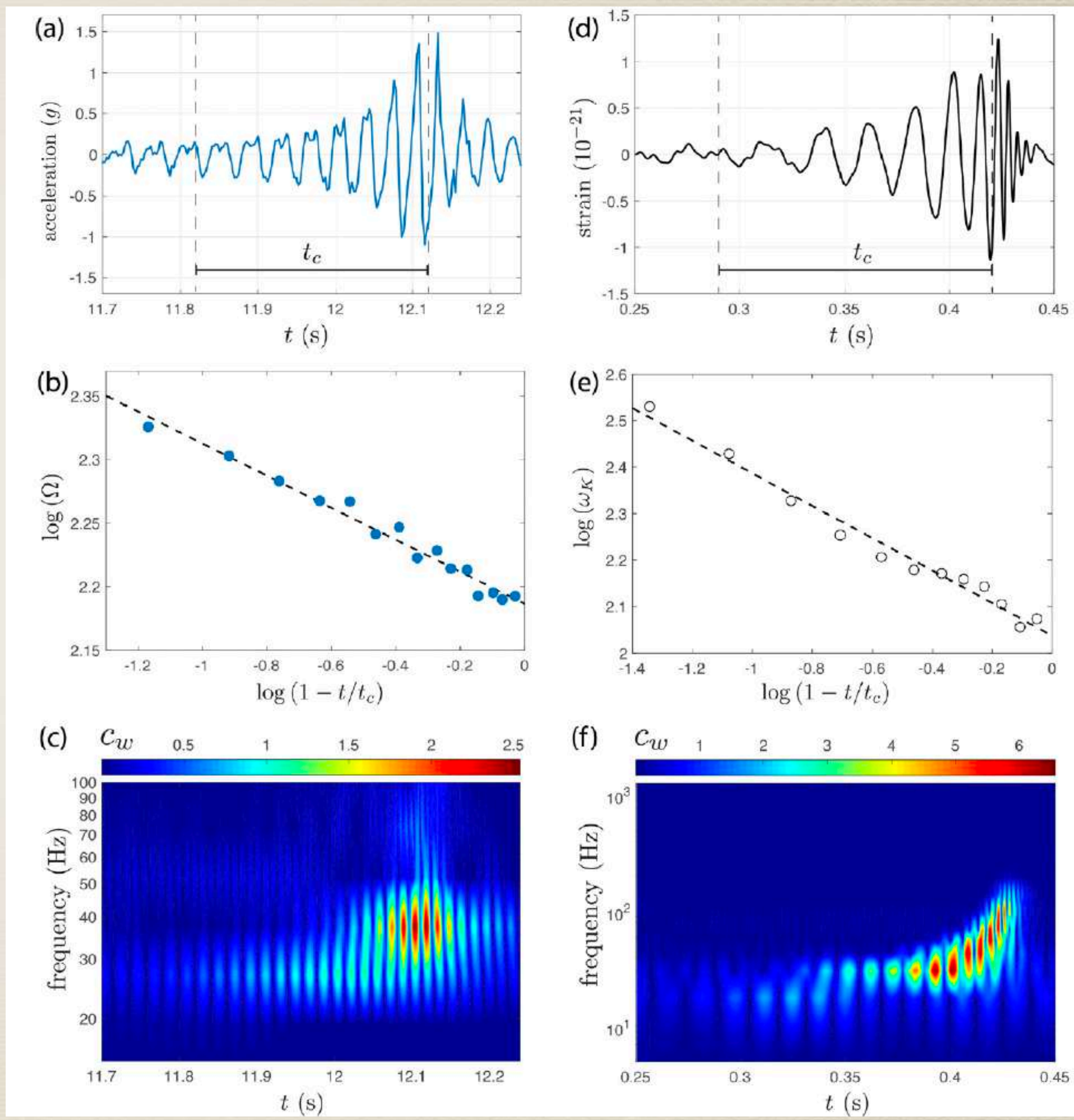
D.D. Meringolo, F. Conidi, A. Mercuri, M. Sposato, R.C. Barberi and G. Pucci, On the analogy between spinning disks coming to rest and merging black holes, American Journal of Physics, 93(7), 551-556 (2025).

Finite Time Singularity of the Euler's Disk

Spinning Disk or Black Holes Merging (Small Science versus Big Science)

Table I. Comparison of the relevant quantities of a disk coming to rest and two black holes merging.

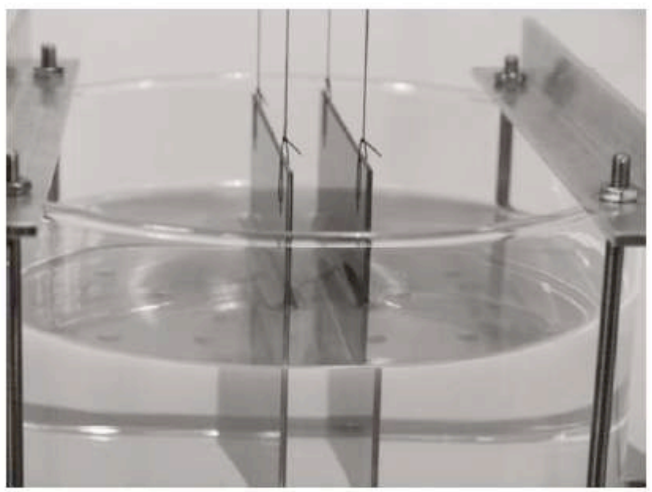
Coming to rest of a spinning disk	Merger of two orbiting black holes
Energy conservation, $dE_{\text{disk}}/dt + P_{\text{diss}} = 0$	Energy conservation, $dE_{\text{orb}}/dt + P_{\text{rad}} = 0$
Disk energy, $E_{\text{disk}} = \frac{3}{2} mgR\alpha$	Orbital energy, $E_{\text{orb}} = -\frac{1}{2} G\mu M/l$
Dissipated power, $P_{\text{diss}} = k\alpha^{-\gamma}$	Gravitational radiation, $P_{\text{rad}} = \frac{32}{5} G^4 \mu^2 M^3 / c^5 l^5$
Coming to rest time, $t_c = 3\alpha_0^{\gamma+1} mgR/2k(\gamma+1)$	Coalescence time, $t_c = \frac{5}{256} c^5 l_0^4 / G^3 \mu M^2$
Contact angle, $\alpha = \alpha_0 (1 - t/t_c)^{1/\gamma+1}$	Orbital separation, $l = l_0 (1 - t/t_c)^{1/4}$
Precession angular velocity, $\Omega = \Omega_0 (1 - t/t_c)^{-1/2(\gamma+1)}$	Orbital angular velocity, $\omega_K = \omega_{K0} (1 - t/t_c)^{-3/8}$

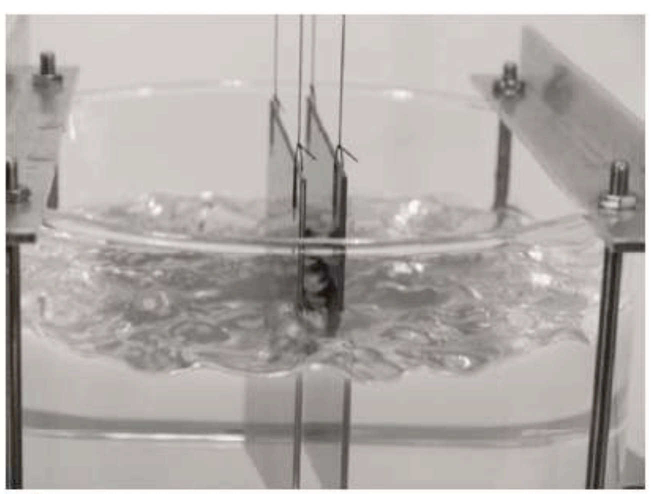


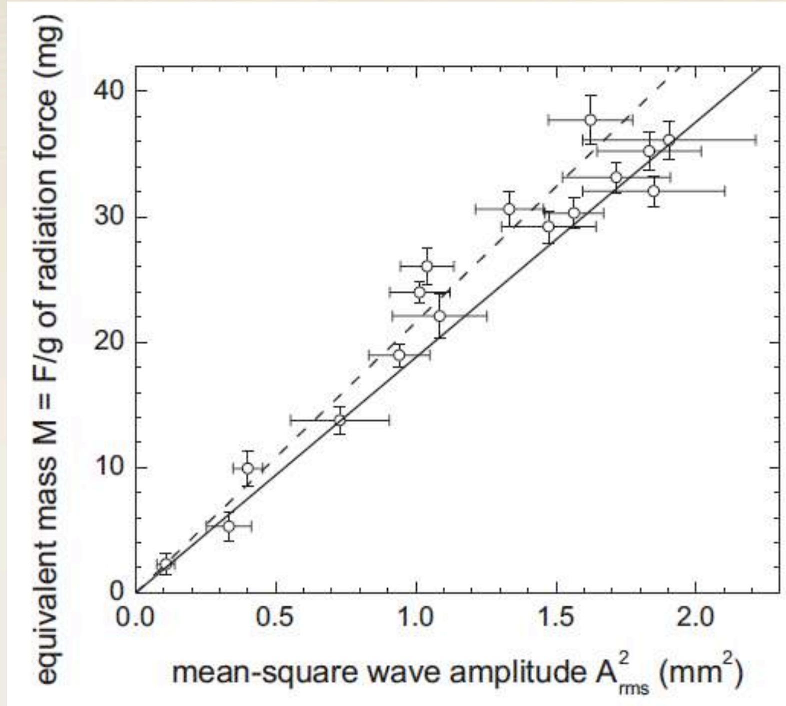
D.D. Meringolo, F. Conidi, A. Mercuri, M. Sposato, R.C. Barberi and G. Pucci, On the analogy between spinning disks coming to rest and merging black holes, American Journal of Physics, 93(7), 551-556 (2025).

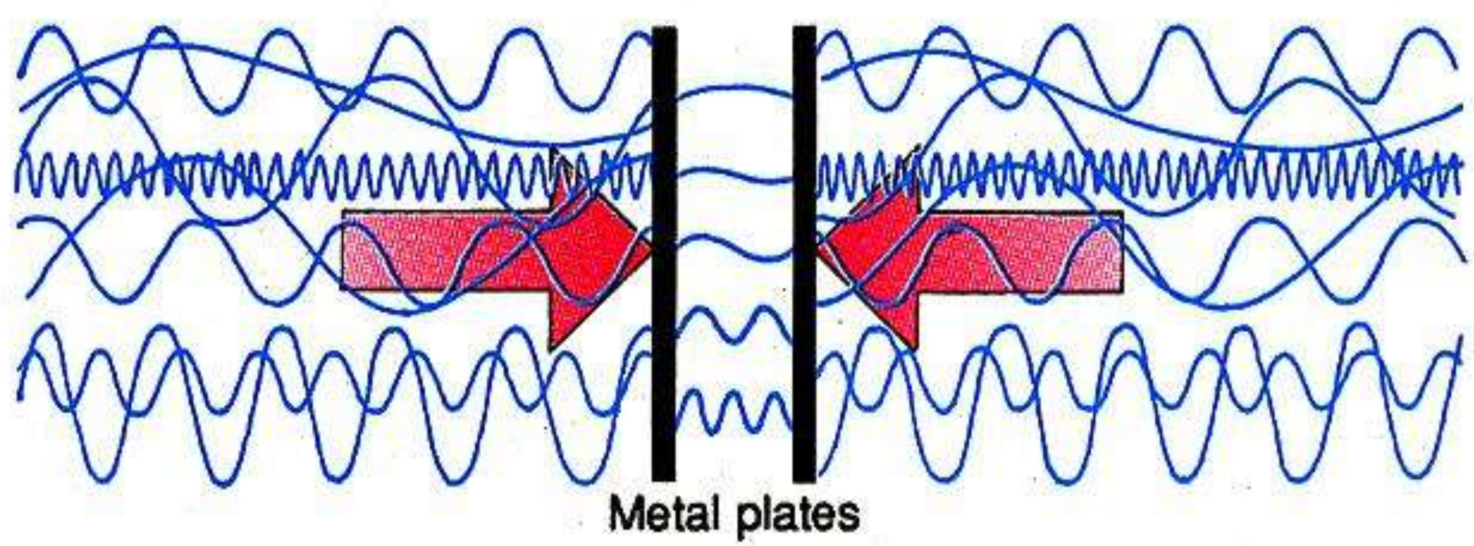
Casimir Effect with Water Waves : Another Quantum Fluctuations Process.


(a)


(b)


(c)





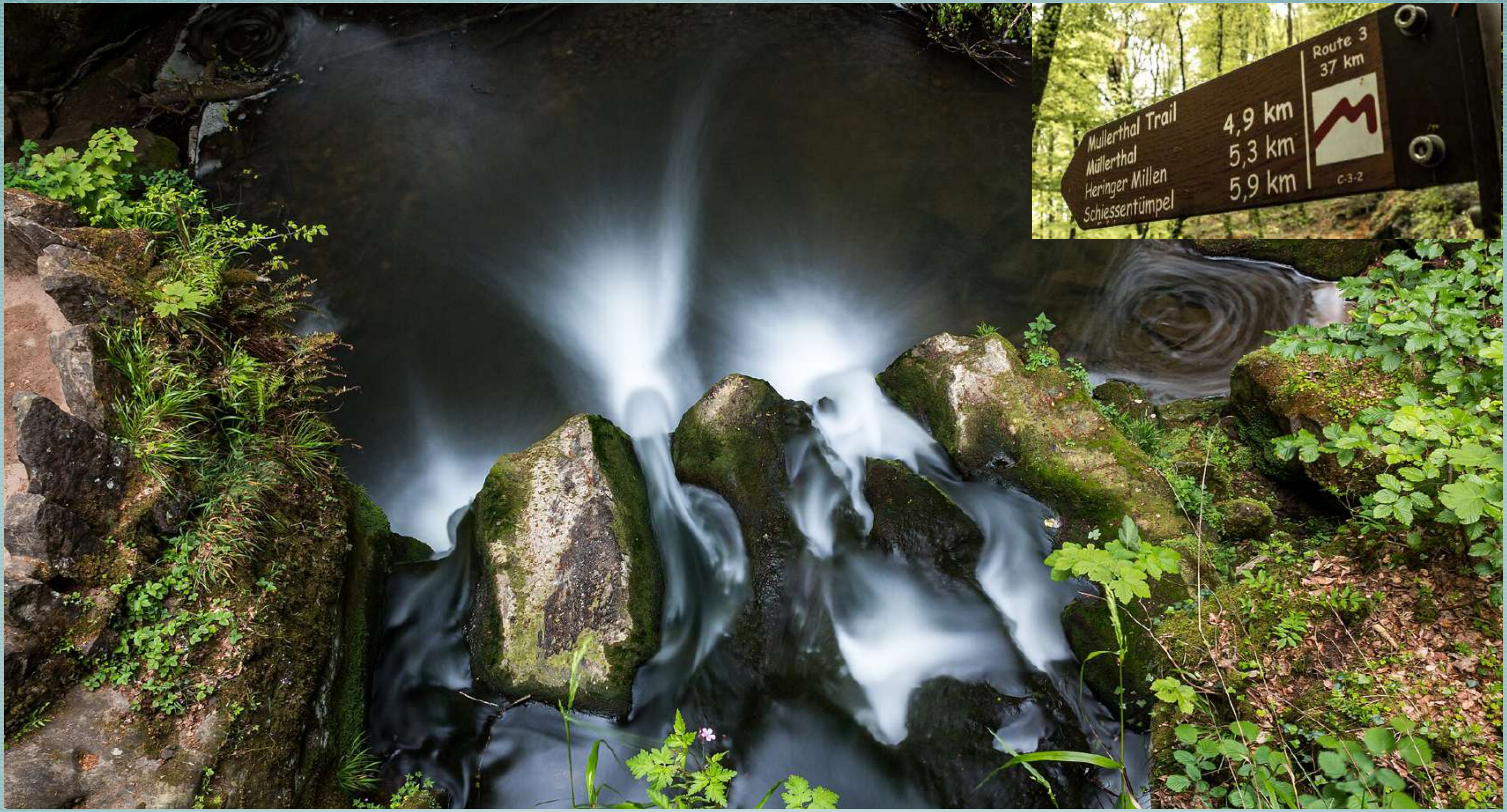
The Casimir effect: an imbalance in the quantum fluctuations of empty space can push two metal plates together

A water wave analog of the Casimir effect

Bruce C. Denardo,[a] Joshua J. Puda, and Andrés Larraza

Am. J. Phys., Vol. 77, No. 12, December 2009

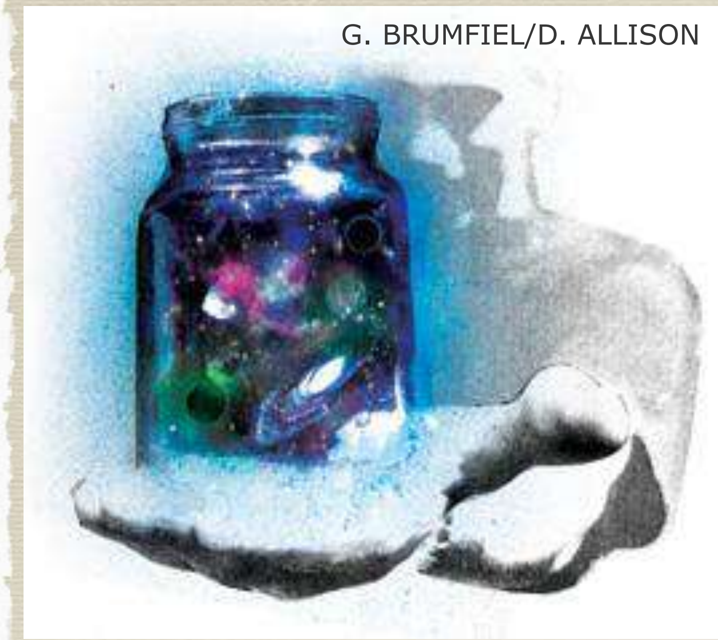
ANALOGUE AQUATIC SPACETIMES



Germain Rousseaux (Research Director CNRS) and Co.,
Team Leader of Curiosity,
Institut Pprime, Futuroscope, Poitiers, (France)

IAS Luxembourg University, Quantum Analogue Workshop,
15th-16th September 2025 Luxembourg

A Warning



Cosmos in a Bottle, Nature, Vol. 451, 17, 236, January 2008.

"Frankly," says Wolfgang Ketterle, a Nobel-Prize-winning condensed-matter physicist at the Massachusetts Institute of Technology in Cambridge, "I don't think a table-top experiment will answer fundamental questions about the cosmos any time soon." "It is really pushing experimental technique to its limits," says Ketterle. Basing a career on such analogies would be "scientific suicide", he says, especially given their tentative link to actual cosmology.

ROSETTA STONE

LP disc	black hole
label	interior of the black hole
playing groove	exterior of the black hole
dead wax	black hole horizon
play hole	central singularity
RPM	Angular momentum
sound speed	light speed
Mister Fahrenheit	Hawking Radiation
supersonic (wo)man	analogue experimentalist

(Don't stop me now)

(Don't stop me)

'Cause I'm having a good time,

having a good time

I'm a shooting star, leaping through the sky

Like a tiger defying the laws of gravity

I'm burnin' through the sky, yeah

Two hundred degrees

That's why they call me Mister Fahrenheit

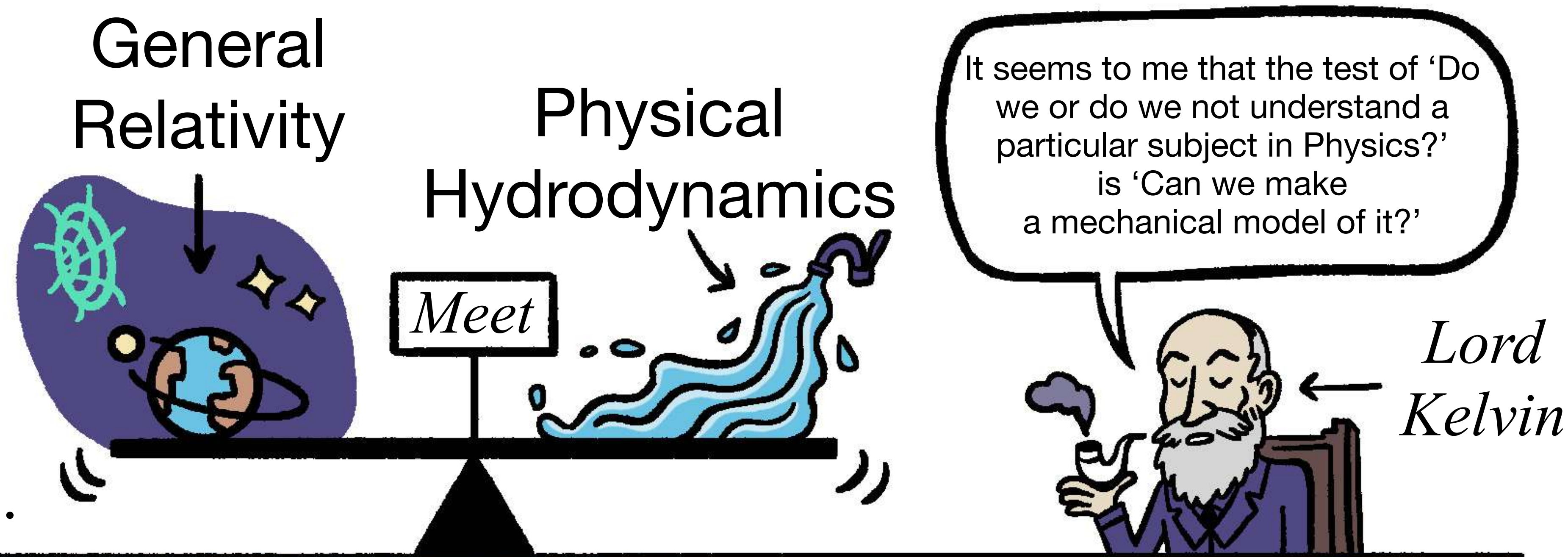
I'm traveling at the speed of light

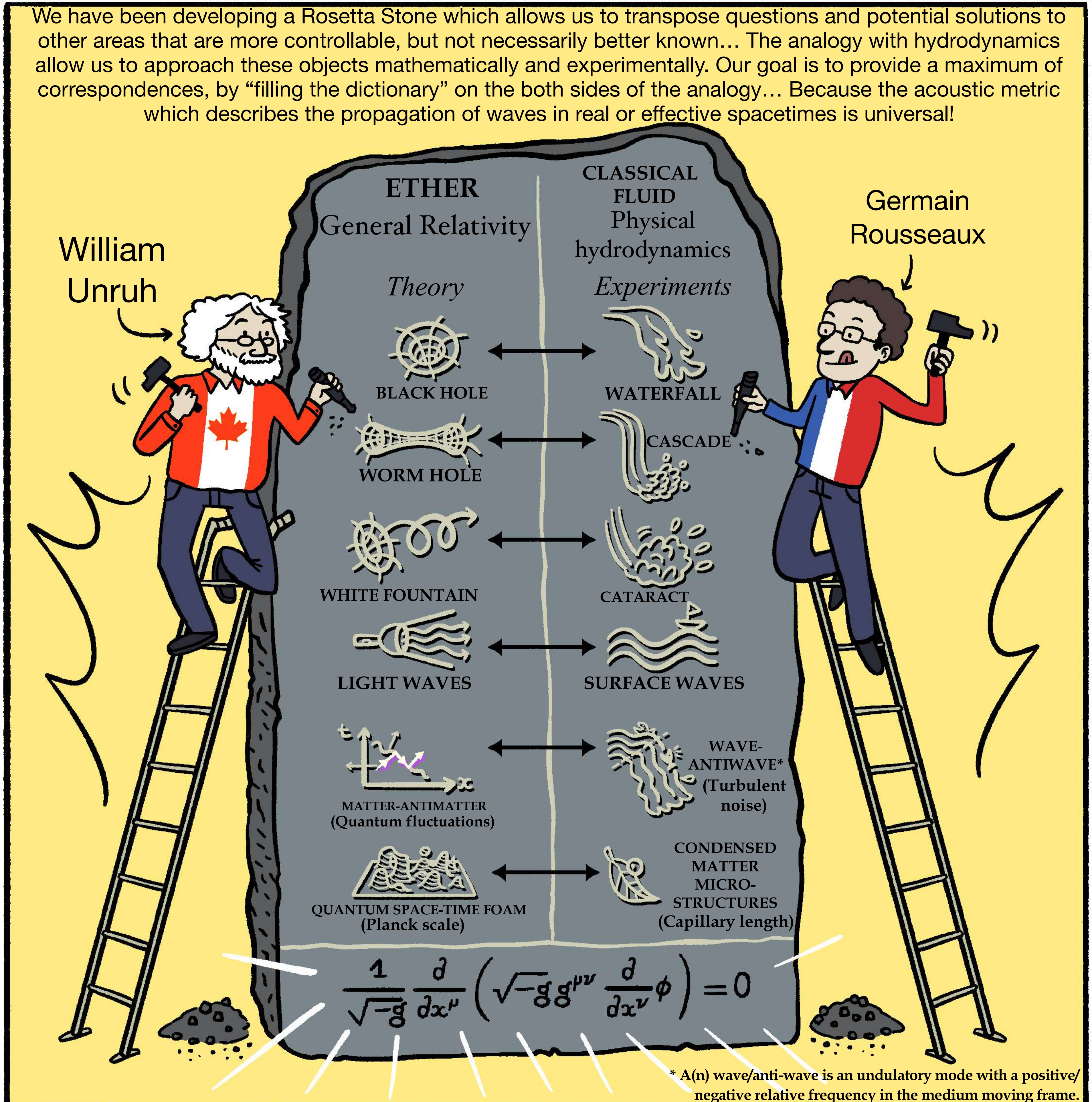
I wanna make a supersonic man out of you

SO, WE PROPOSE…

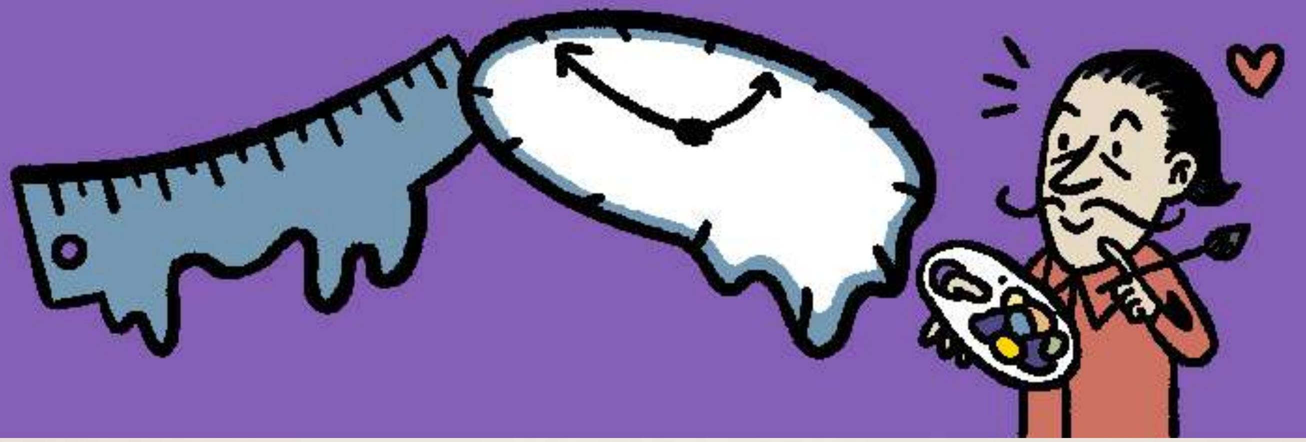
Analogue Gravity & Cosmology

Where…




We have been developing a Rosetta Stone which allows us to transpose questions and potential solutions to other areas that are more controllable, but not necessarily better known… The analogy with hydrodynamics allow us to approach these objects mathematically and experimentally. Our goal is to provide a maximum of correspondences, by "filling the dictionary" on the both sides of the analogy… Because the acoustic metric which describes the propagation of waves in real or effective spacetimes is universal!
William Unruh
Germain Rousseaux
ETHER
General Relativity
Theory
CLASSICAL FLUID
Physical hydrodynamics
Experiments
BLACK HOLE
WATERFALL
WORM HOLE
CASCADE
WHITE FOUNTAIN
CATARACT
LIGHT WAVES
SURFACE WAVES
MATTER-ANTIMATTER (Quantum fluctuations)
WAVE-ANTIWAVE* (Turbulent noise)
QUANTUM SPACE-TIME FOAM (Planck scale)
CONDENSED MATTER MICRO-STRUCTURES (Capillary length)
$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}\,g^{\mu\nu}\frac{\partial}{\partial x^\nu}\phi\right)=0$
* A(n) wave/anti-wave is an undulatory mode with a positive/negative relative frequency in the medium moving frame.

The spatio-temporal dependencies are synthesized in the concept of metric which underlies our measurement standards (rulers and clocks).

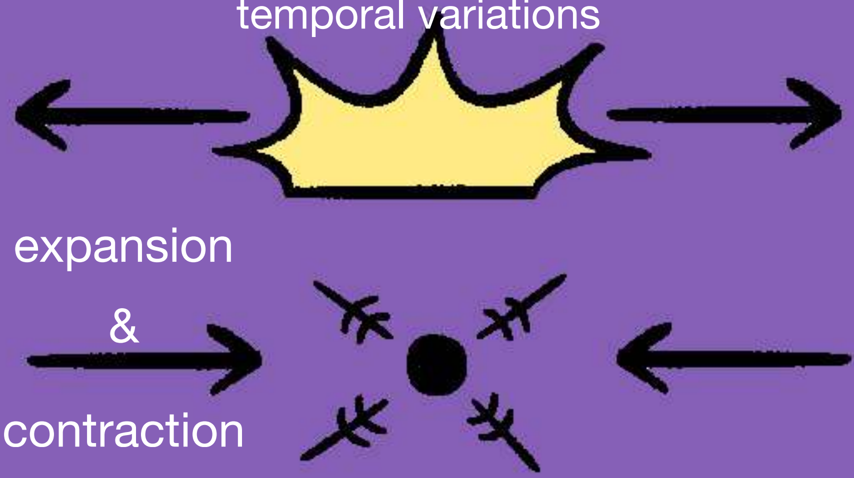


Acoustic Metric whose determinant is g

$$g_{\mu\nu} = \Omega \left(\begin{array}{c|c} -(c^2 - v^2) & -v^T \\ \hline -v & I \end{array} \right)$$

c is the wave speed / v is the flow speed

With the corresponding temporal variations



expansion

&

contraction

Acoustic Rays as Geodesics in an Effective Space-Time (R. White 1973)

dx/dt: speed of the wavefront

c: wave speed (acoustic, water waves, phonons in a BEC)

v: flow/medium speed

$\hat{n}$: normal vector of norm equal to one

ds: space-time interval

u: column four-vector

u^T: line four vector

$g_{\mu\nu}$: acoustic metric

$$\frac{d\mathbf{x}}{dt} = c\hat{\mathbf{n}} + \mathbf{v}.$$

$$\hat{\mathbf{n}} = \frac{1}{c}\left[\frac{d\mathbf{x}}{dt} - \mathbf{v}\right]$$

$$1 = \frac{1}{c^2}\left[\frac{d\mathbf{x}}{dt} - \mathbf{v}\right] \cdot \left[\frac{d\mathbf{x}}{dt} - \mathbf{v}\right] = \frac{1}{c^2}\left[\left(\frac{d\mathbf{x}}{dt}\right)^2 - 2\mathbf{v} \cdot \frac{d\mathbf{x}}{dt} + \mathbf{v}^2\right]$$

$$c^2 dt^2 = d\mathbf{x}^2 - 2(\mathbf{v} \cdot d\mathbf{x})\, dt + \mathbf{v}^2 dt^2$$

$$-(c^2 - \mathbf{v}^2)\, dt^2 - 2(\mathbf{v} \cdot d\mathbf{x})\, dt + d\mathbf{x} \cdot d\mathbf{x} = 0$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$\mathbf{u}^T \equiv (dx^\mu)^T = (dt, d\mathbf{x})$$

$$(dt, d\mathbf{x}) \begin{pmatrix} -(c^2 - \mathbf{v}^2)dt - (\mathbf{v} \cdot d\mathbf{x}) \\ -\mathbf{v}dt + d\mathbf{x} \end{pmatrix} = 0.$$

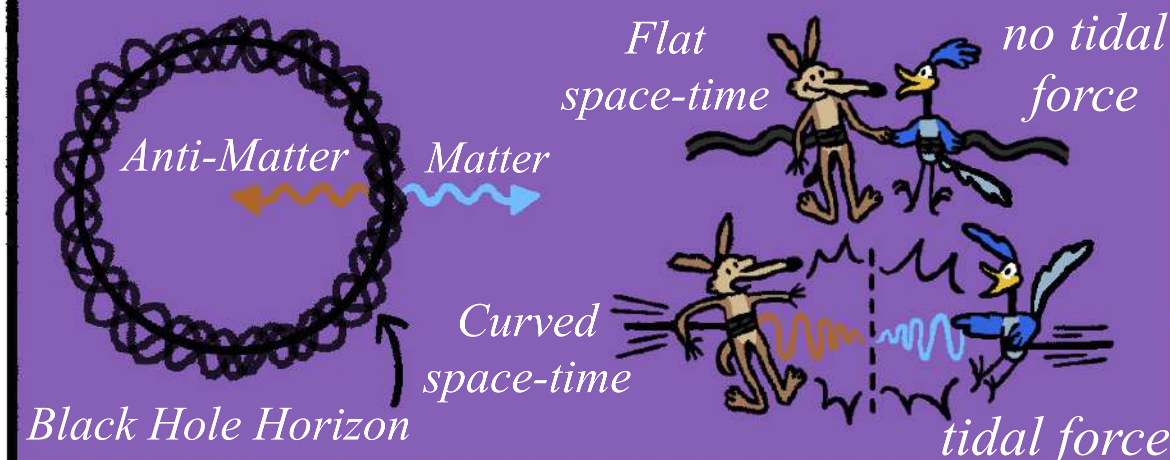
$$(dt, d\mathbf{x}) \begin{pmatrix} -(c^2 - \mathbf{v}^2) & -\mathbf{v}^T \\ -\mathbf{v} & \mathbb{I}_{3\times 3} \end{pmatrix} \begin{pmatrix} dt \\ d\mathbf{x} \end{pmatrix} = 0.$$

$$g_{\mu\nu} = \Omega^2 \begin{pmatrix} -(c^2 - \mathbf{v}^2) & -\mathbf{v}^T \\ -\mathbf{v} & \mathbb{I}_{3\times 3} \end{pmatrix}$$

QUESTIONS WE ARE WORKING ON 1

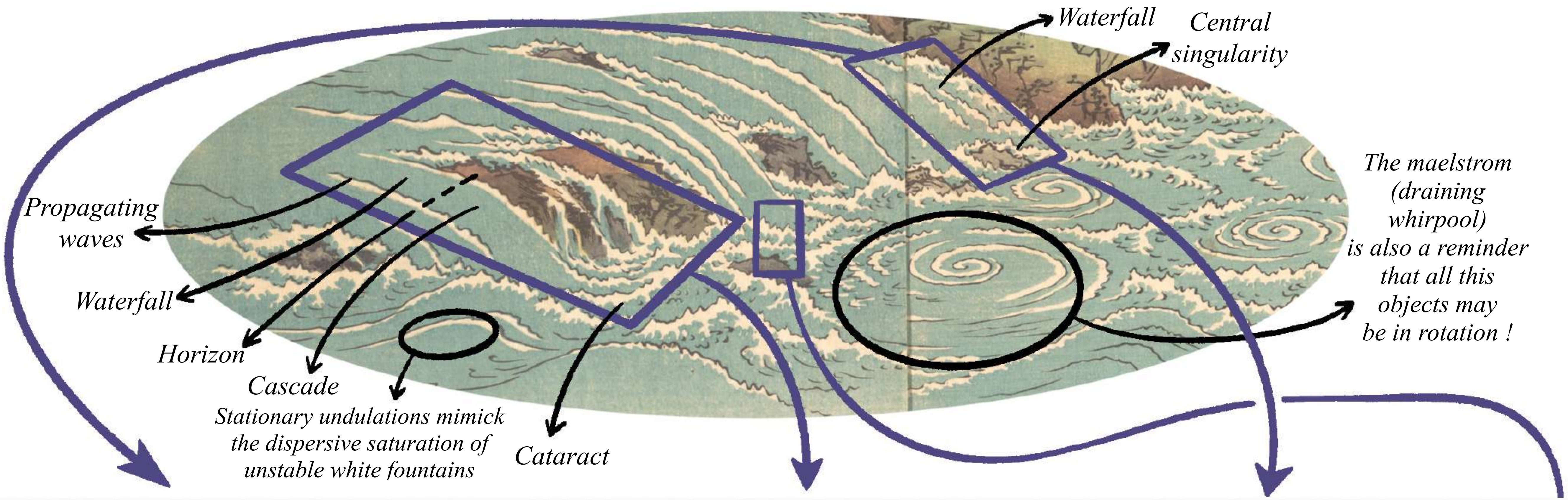
WHAT IS HAPPENING INSIDE A BLACK HOLE?

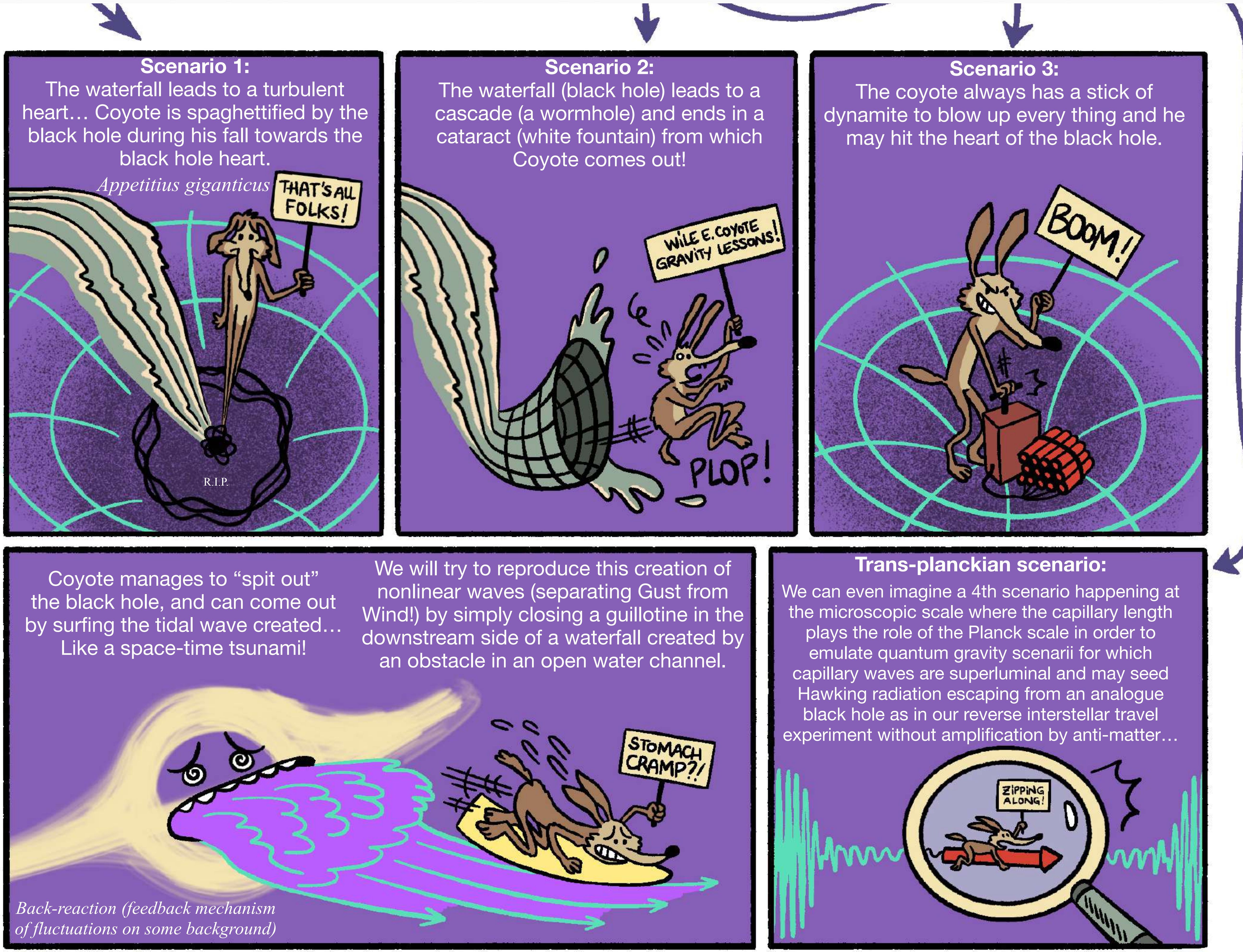
Vacuum fluctuates: virtual pairs of particles and anti-particles become real when polarized by an external field (electric/gravity field according to Schwinger/Hawking, an analogy due to Gribov). A horizon separates the pairs and the escaping mode (particle or wave) is amplified by the absorption of the anti-mode (anti-particle or anti-wave).



If we personify these particles, in a duo of linked characters, like Road Runner (BipBip in French) and Wile E. Coyote, we can develop 3 **classical** and 1 **quantum-like** scenarii regarding the fate of these particles in the vicinity of a black hole. BipBip always escapes (when the antiparticles appear, the particles extract energy of the black hole!), but what will happen to Wile E. Coyote?

Interestingly, in this extract from this Hiroshige woodblock print (Whirlpools of Naruto Straits in Awa Province, in the 19th century), we have a perfect analogous representation of the 4 scenarii describing one's fate within a black hole…




Scenario 1:
The waterfall leads to a turbulent heart… Coyote is spaghettified by the black hole during his fall towards the black hole heart.
Appetitius giganticus
THAT'S ALL FOLKS!
R.I.P.
Scenario 2:
The waterfall (black hole) leads to a cascade (a wormhole) and ends in a cataract (white fountain) from which Coyote comes out!
WILE E. COYOTE GRAVITY LESSONS!
PLOP!
Scenario 3:
The coyote always has a stick of dynamite to blow up every thing and he may hit the heart of the black hole.
BOOM!
Coyote manages to “spit out” the black hole, and can come out by surfing the tidal wave created… Like a space-time tsunami!
We will try to reproduce this creation of nonlinear waves (separating Gust from Wind!) by simply closing a guillotine in the downstream side of a waterfall created by an obstacle in an open water channel.
STOMACH CRAMP?!
Back-reaction (feedback mechanism of fluctuations on some background)
Trans-planckian scenario:
We can even imagine a 4th scenario happening at the microscopic scale where the capillary length plays the role of the Planck scale in order to emulate quantum gravity scenarii for which capillary waves are superluminal and may seed Hawking radiation escaping from an analogue black hole as in our reverse interstellar travel experiment without amplification by anti-matter…
ZIPPING ALONG!

A Hydraulic Black Hole in the Morbihan Gulf (France)

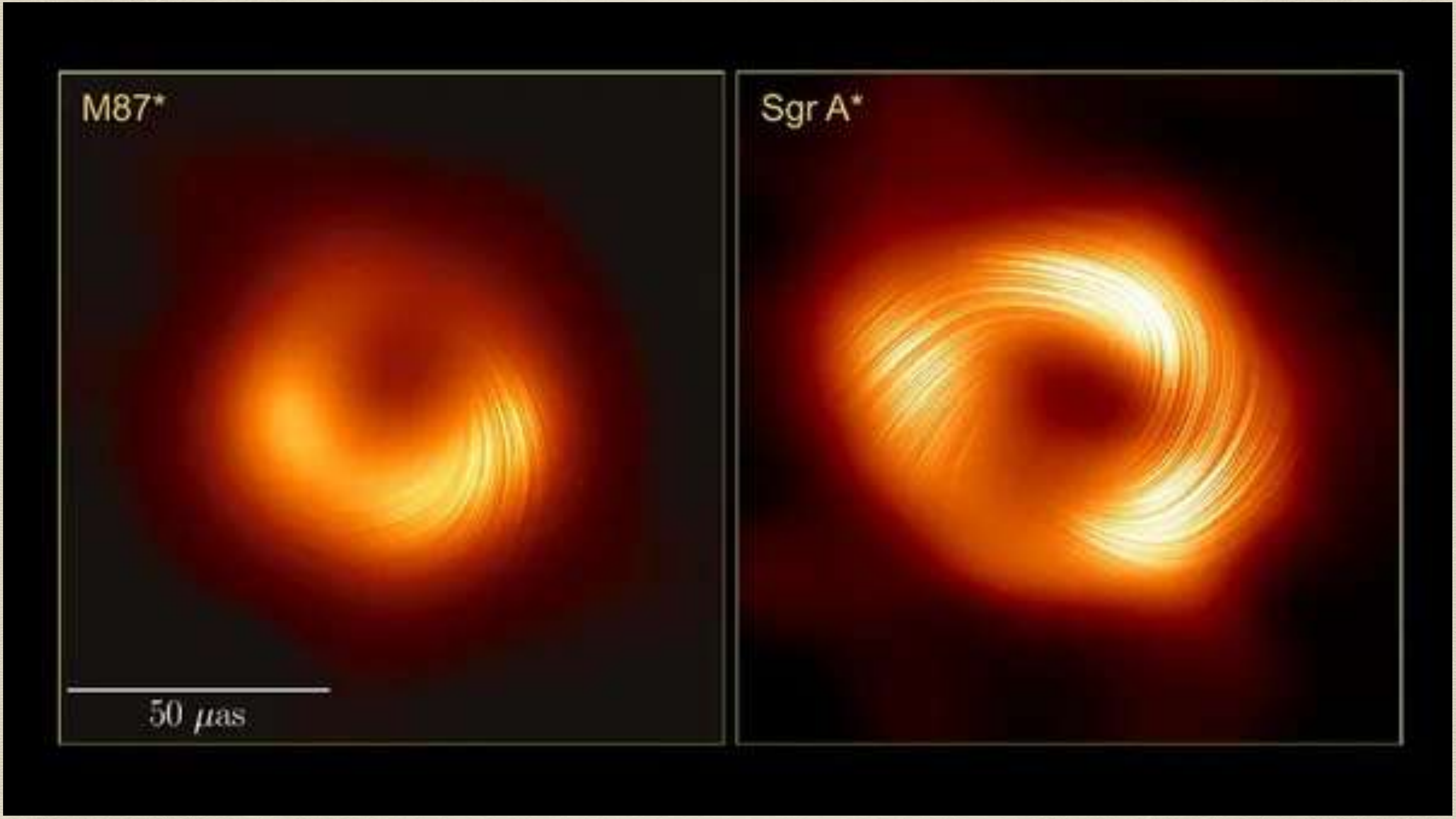
Gois of Berder Island, rising tide : « Pascalian » black hole by water depths mismatch.

2D Hydraulic Black Hole

The Spaghettification in a Natural Hydraulic Black Hole

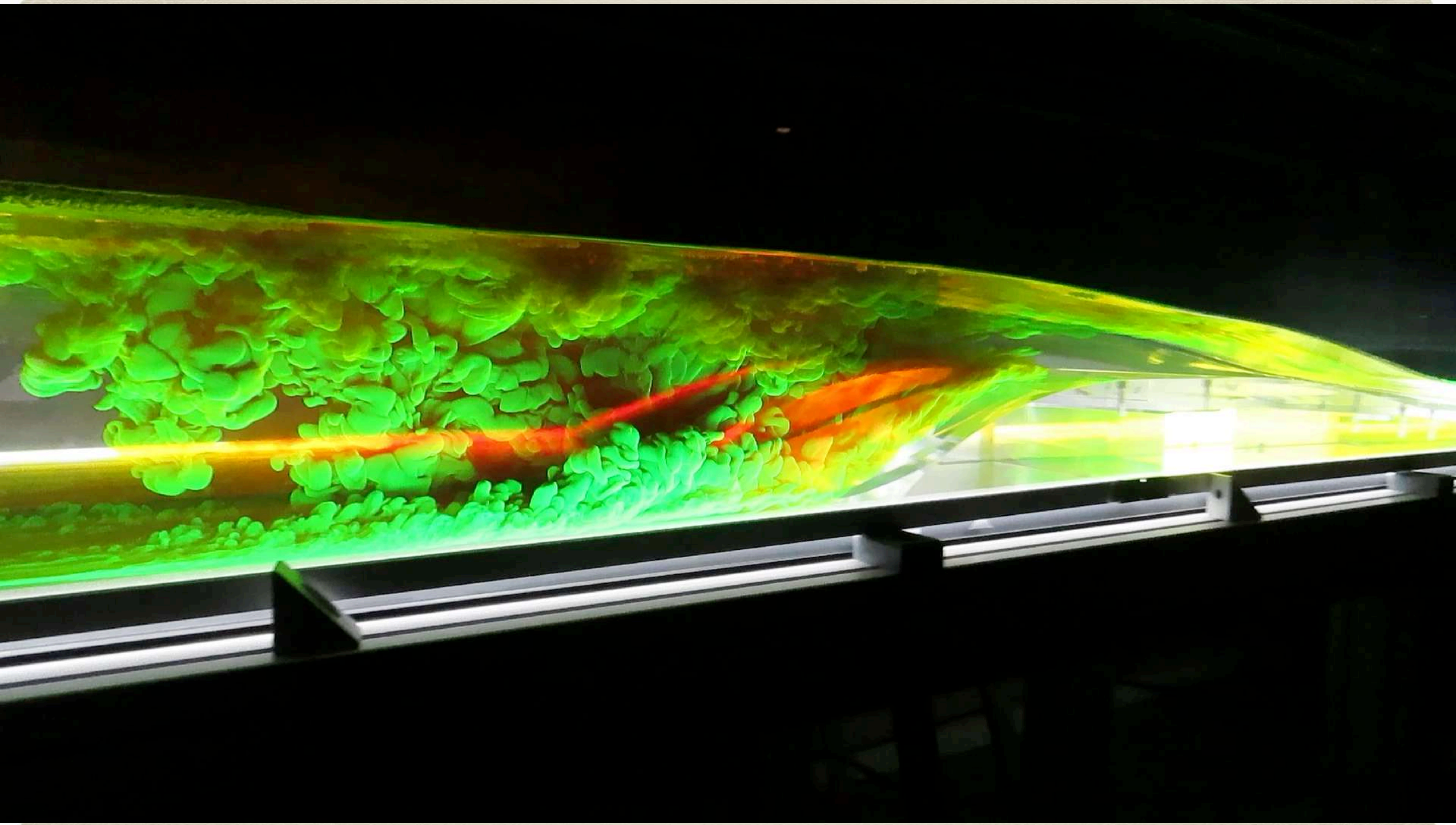
The Spaghettification in an Astrophysical Black Hole

Introducing the Actual Black Holes



Courtesy Event Horizon Telescope (EHT) 2024

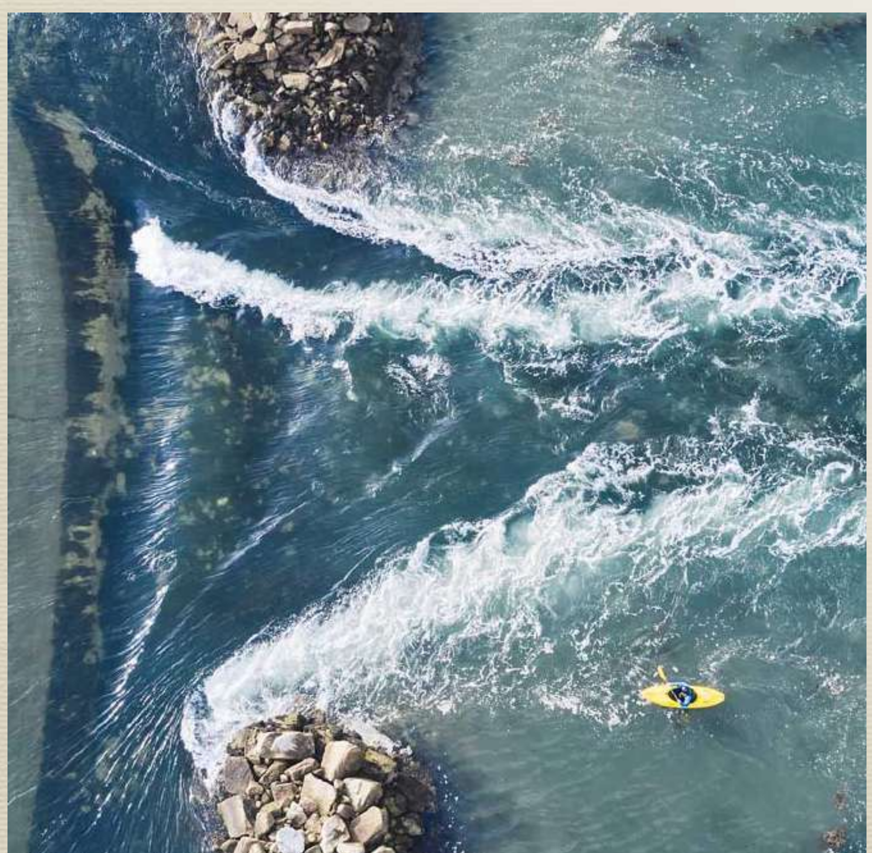
An Aquatic Black Hole Eating Matter

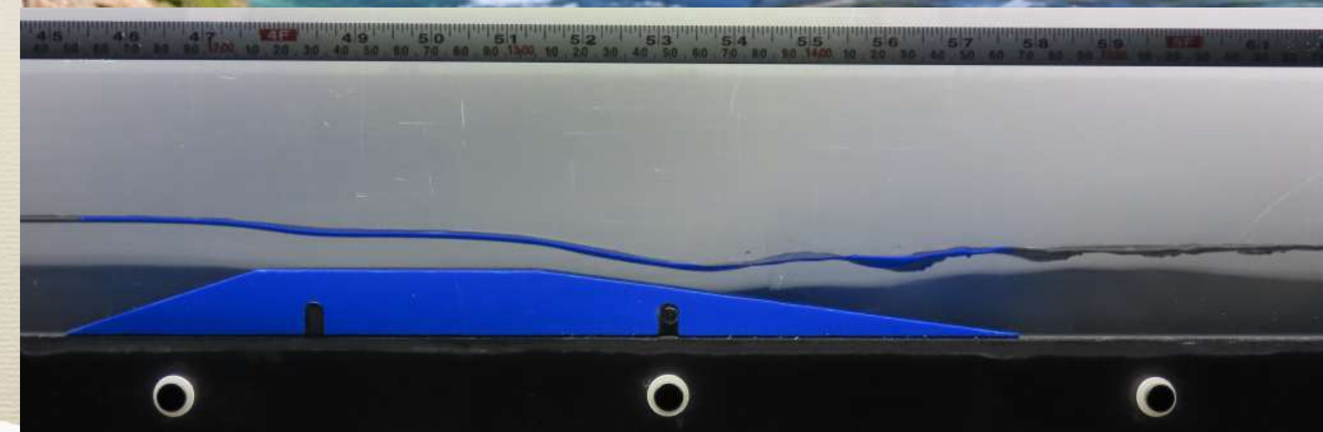


2D Hydraulic Black Hole and White Fountain

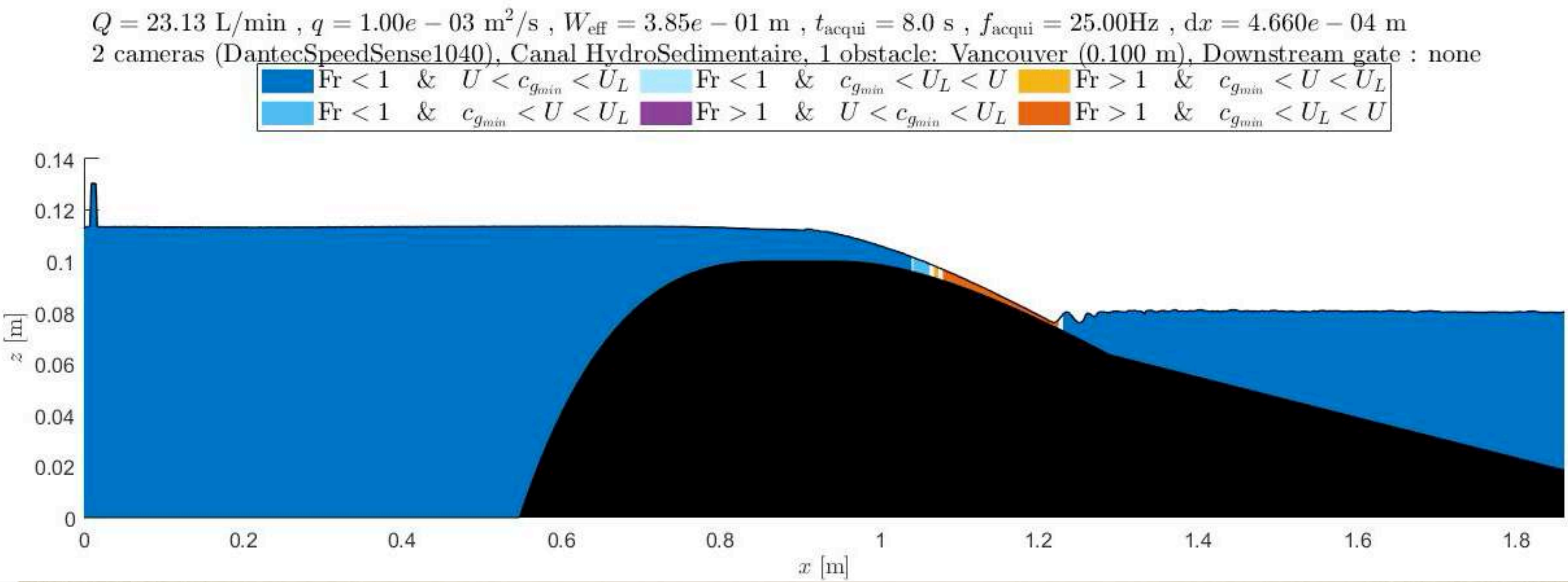
small tidal range
small upstream
water depth
flood tide

Transcritical accelerating
non-dispersive waterfall followed
by a transcritical decelerating
dispersive undulation





Filling up the Water Channel (or the Bathtub...)



Alexis Bossard,

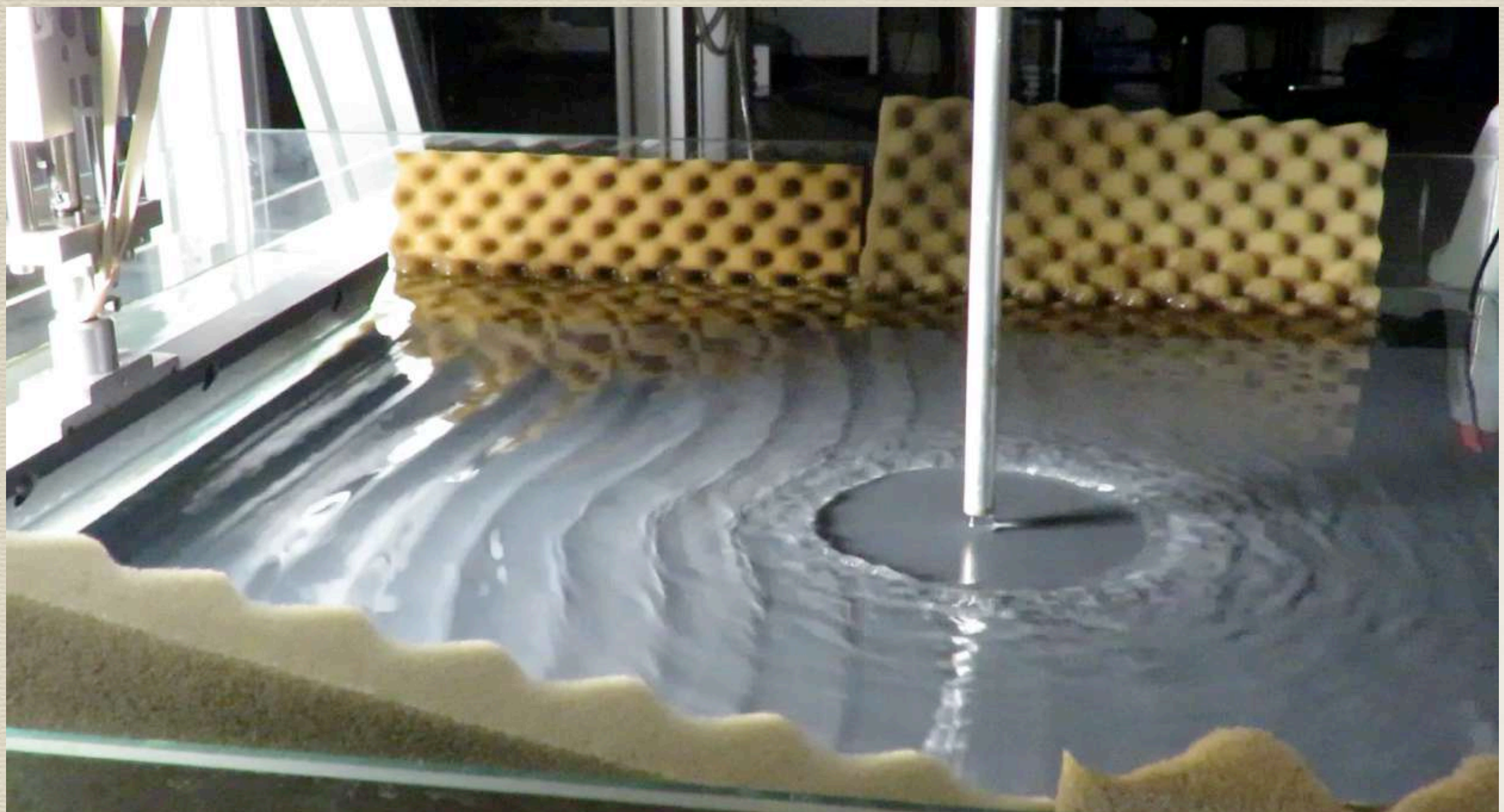
PhD Thesis to be defended in

december 2025

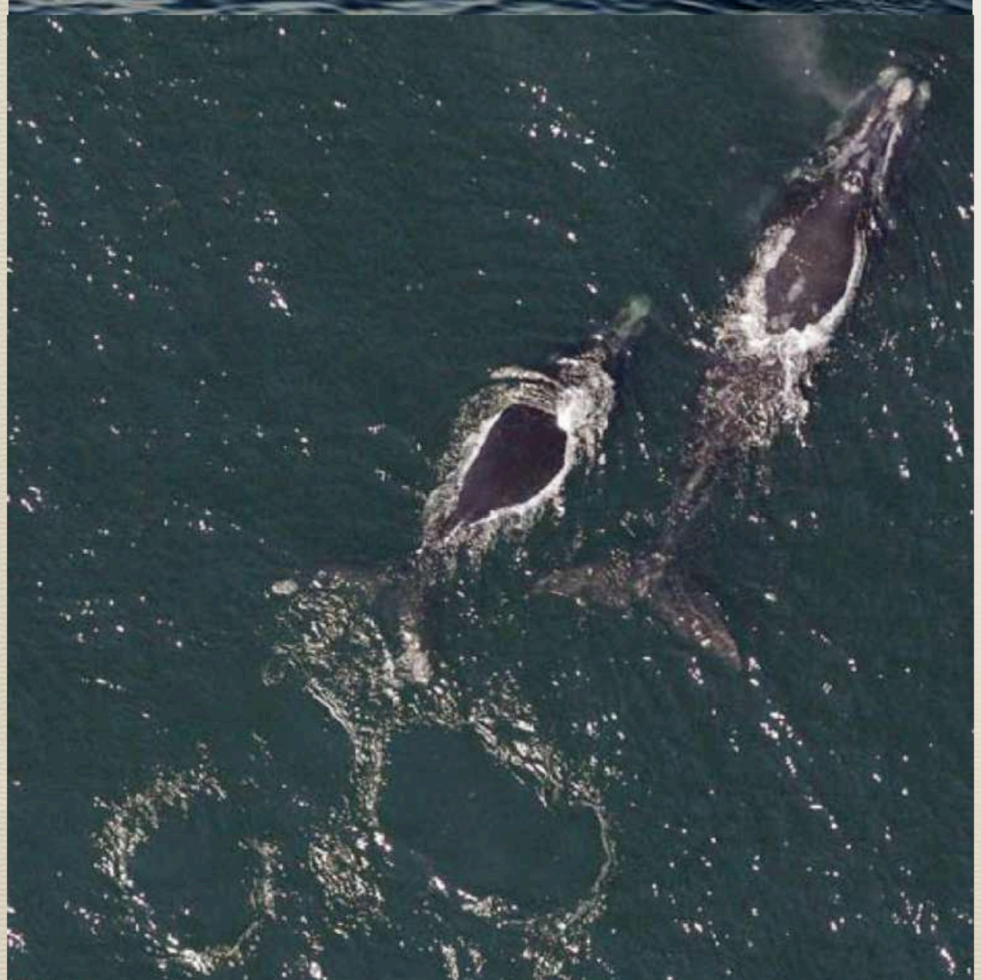
Medium Water Channel of the

Pprime Institute (France)

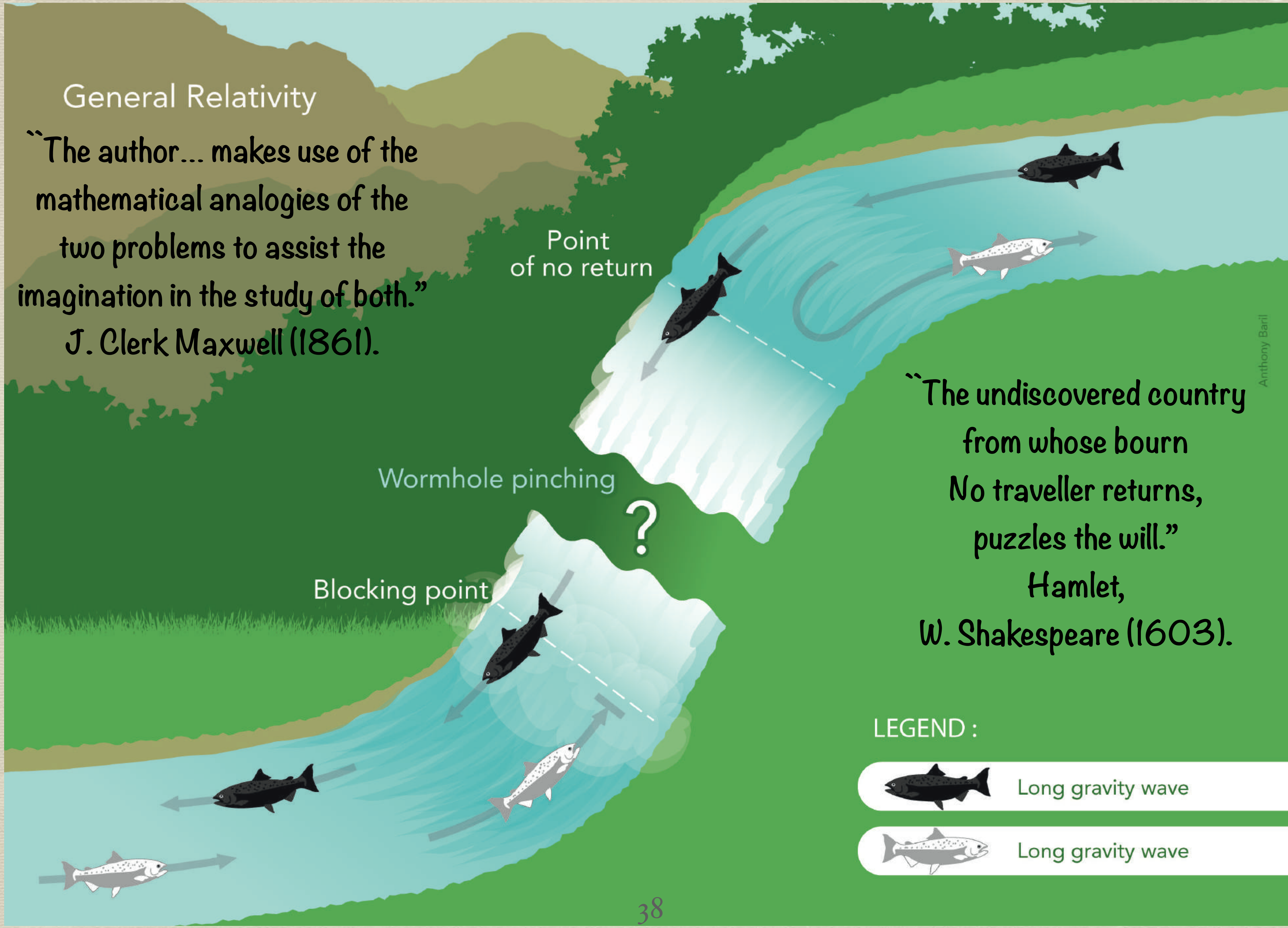
The Circular Jump as a 2D Hydraulic White Fountain



The Whale Flukeprint as a Dispersive White Hole Horizon (no metric)



1D Black Hole, White Fountain and Wormhole in General Relativity

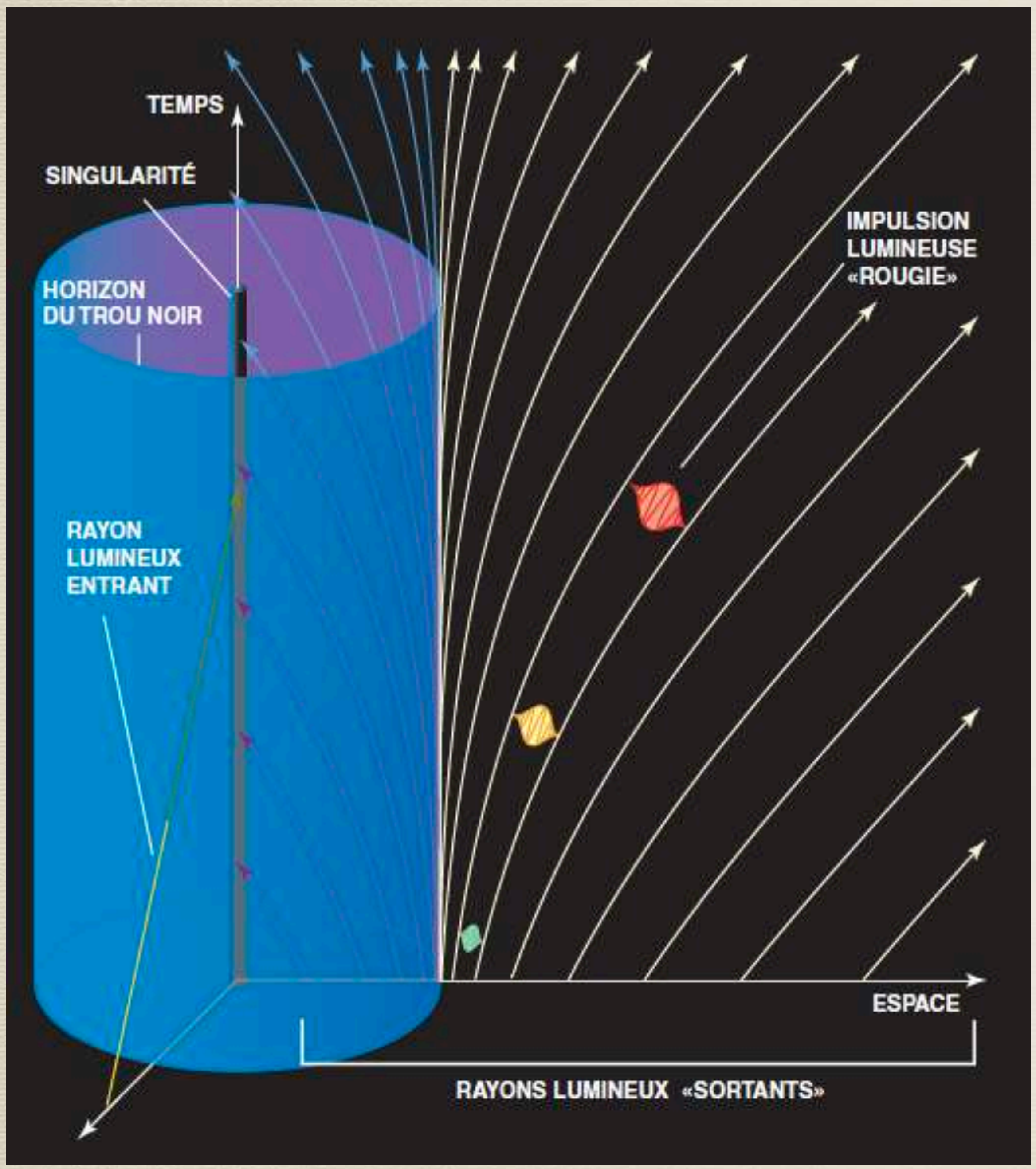


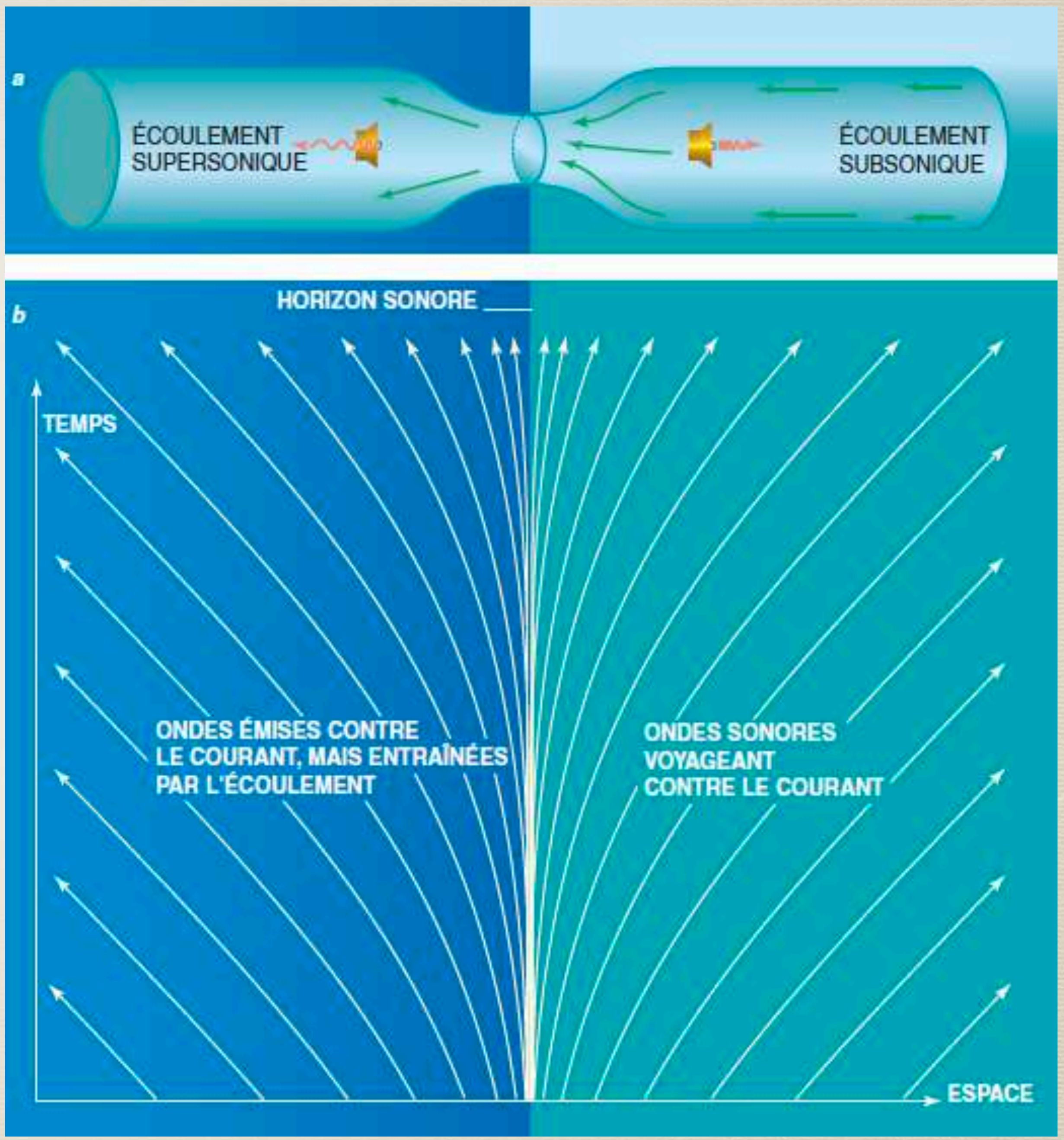
"The author... makes use of the mathematical analogies of the two problems to assist the imagination in the study of both."
J. Clerk Maxwell (1861).

"The undiscovered country
from whose bourn
No traveller returns,
puzzles the will."
Hamlet,
W. Shakespeare (1603).

Spacetime Diagrams (a Kinematic Analogy)

A Black Hole in a P-G Spacetime A Dumb Hole in a De Laval Nozzle





$$\frac{dx}{dt} = v(x) + c(x) \simeq \kappa x \quad \frac{dx}{\kappa x} \simeq dt \quad t \simeq t_0 + \frac{1}{\kappa} ln(\frac{x}{x_0}) \quad \begin{array}{c} t \to \pm\infty \\ x \to 0^{\pm} \end{array}$$

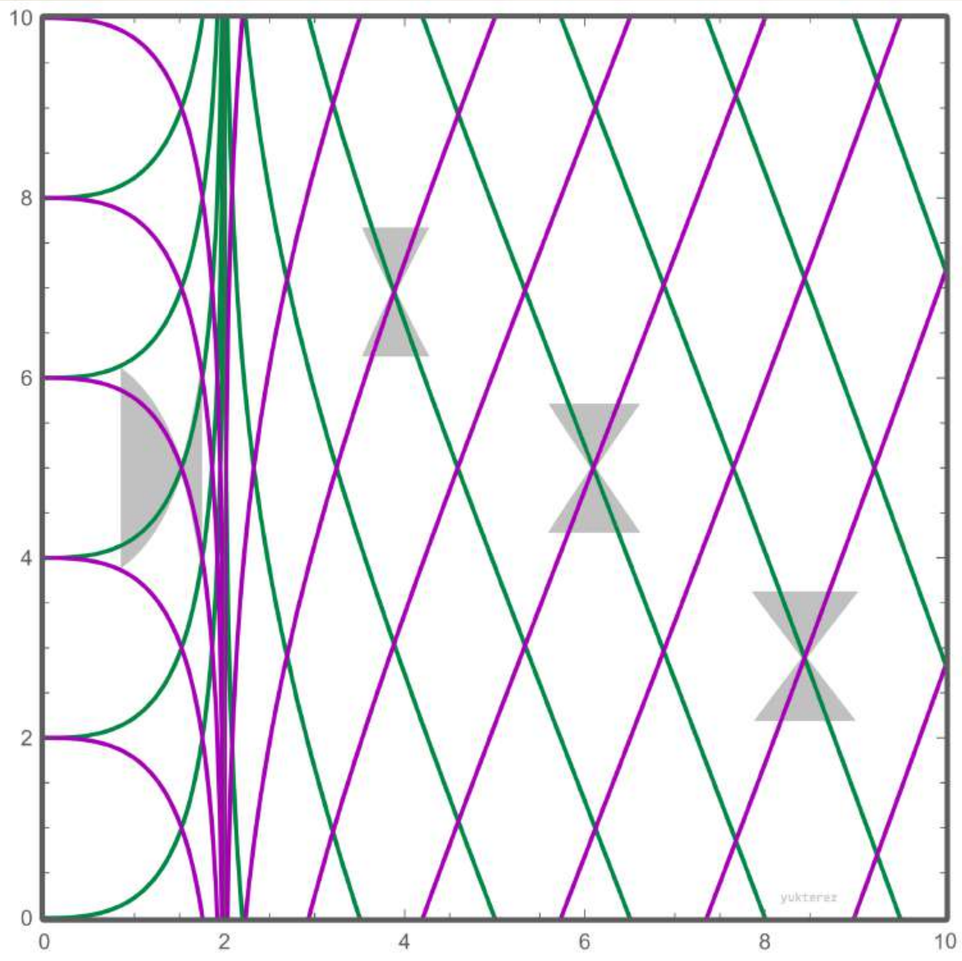
Courtesy Renaud Parentani (Pour La Science, 2022).

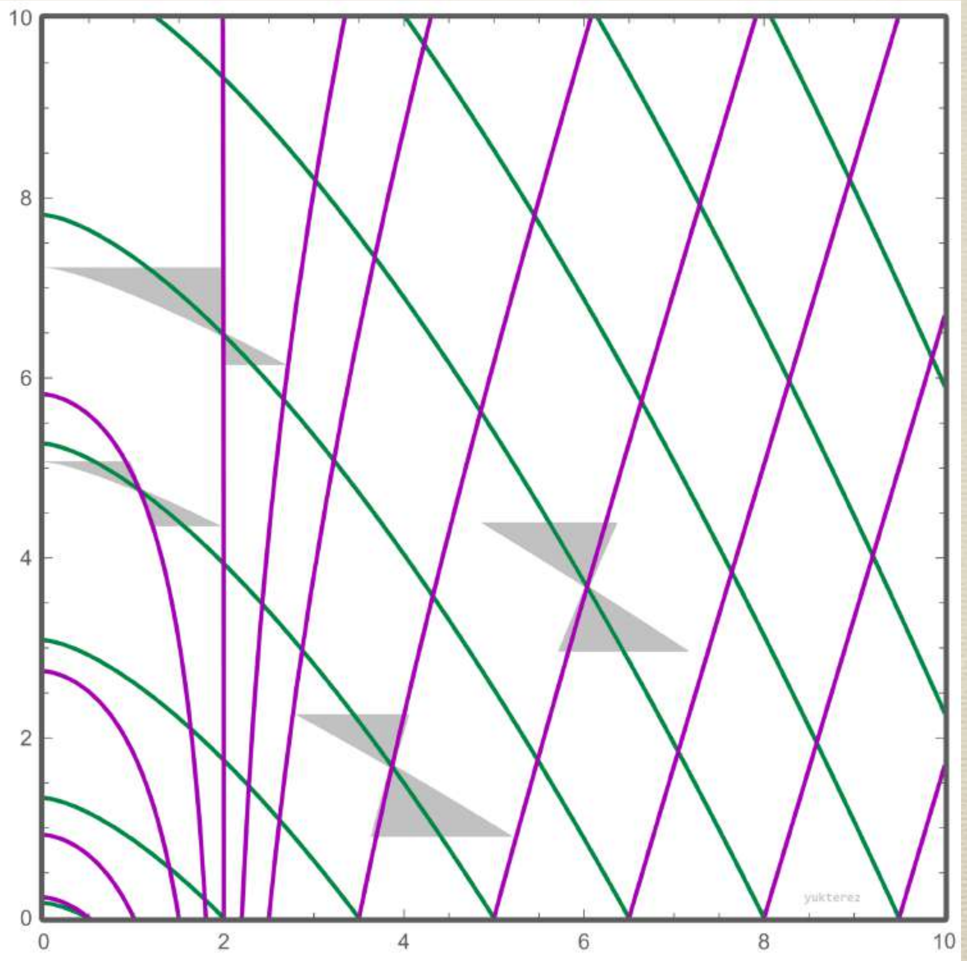
Schwarzschild-Droste versus Painlevé-Gullstrand Space-times

« Painlevé wrote to Einstein to introduce his solution and invited Einstein to Paris for a debate. In Einstein's reply letter (December 7), he apologized for not being in a position to visit soon and explained why he was not pleased with Painlevé's arguments, emphasising that the coordinates themselves have no meaning. Finally, Einstein came to Paris in early April. On the 5th of April 1922, in a debate at the "Collège de France" with Painlevé, Becquerel, Brillouin, Cartan, De Donder, Hadamard, Langevin and Nordmann on "the infinite potentials", Einstein, baffled by the non quadratic cross term in the line element, rejected the Painlevé solution. » J. Fric

« The "trick" of the Painlevé proposal was that he no longer stuck to a full quadratic (static) form but instead, allowed a cross time-space product making the metric form no longer static but stationary and no longer direction symmetric but preferentially oriented. » J. Fric

P-G metric=acoustic metric in AG





Paul Painlevé, La mécanique classique et la théorie de la relativité, C. R. Acad. Sci. (Paris) 173, p. 677-680 (1921).

Allvar Gullstrand, Allgemeine Lösung des statischen Einkörperproblems in der Einsteinschen Gravitationstheorie, Arkiv för Matematik, Astronomi och Fysik, 16 (8), p. 1-15 (1922).

https://astromontgeron.fr/Painleve-article-english.pdf

First Identification of Wave Physics in an Effective Metric and of a Condensed Matter Hawking Temperature

The basic idea: consider fluid flow – Unruh (1981, 1995), Visser (1998)

Continuity: $$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{v}) = 0$$

Euler's equation: $$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{\nabla p}{\rho} + \mathbf{F}$$

Assume fluid is irrotational ($\mathbf{v} = \nabla \phi$), inviscid and barotropic ($p = p(\rho)$) and linearize:

$$\rho \to \rho_0 + \rho_1 \qquad \phi \to \phi_0 + \phi_1 \qquad p \to p_0 + p_1$$

$$0 = -\partial_t \left(\frac{\partial \rho}{\partial p} \rho_0 (\partial_t \psi_1 + \mathbf{v_0} . \nabla \psi_1) \right) + \nabla . \left(\rho_0 \nabla \psi_1 - \frac{\partial \rho}{\partial p} \rho_0 \boldsymbol{v}_0 (\partial_t \psi_1 + \mathbf{v_0} . \nabla \psi_1) \right)$$

First Identification of Wave Physics in an Effective Metric and of a Condensed Matter Hawking Temperature

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$$\rho \rightarrow \rho_0 + \rho_1 \qquad \phi \rightarrow \phi_0 + \phi_1 \qquad p \rightarrow p_0 + p_1$$

Relativistic wave equation:

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left(\sqrt{-g}\, g^{\mu\nu} \frac{\partial}{\partial x^\nu} \phi_1 \right) = 0$$

with acoustic metric for massless scalar field:

where $$g_{\mu\nu} = \frac{\rho_0}{c} \left(\begin{array}{c|c} -(c^2 - v^2) & -\mathbf{v}^T \\ \hline -\mathbf{v} & \mathbf{I} \end{array} \right) \qquad g = [\det(g^{\mu\nu})]^{-1}$$

The Metric in the Dispersion Relation

Doppler shifted phonon spectrum in moving superfluid and BEC

review:
Barcelo, Liberati & Visser,
Analogue Gravity
Living Rev. Rel. **8** (2005) 12

c speed of sound

$\mathbf{v}_s$ superfluid velocity

$$\boxed{E = cp + \mathbf{p}\cdot\mathbf{v}_s}$$

move $\mathbf{p}\cdot\mathbf{v}_s$ *to the left*

$$\boxed{E - \mathbf{p}\cdot\mathbf{v}_s = cp}$$

take square

$$\boxed{(E - \mathbf{p}\cdot\mathbf{v}_s)^2 - c^2 p^2 = 0} \longrightarrow \boxed{g^{\mu\nu} p_\mu p_\nu = 0} \qquad p_\nu = (-E,\ \mathbf{p})$$

$$g^{00} = -1 \qquad g^{0i} = -v_s^{\,i} \qquad g^{ij} = c^2\delta^{ij} - v_s^{\,i} v_s^{\,j}$$

effective metric

inverse metric $g_{\mu\nu}$ determines effective spacetime
in which phonons move along geodesic curves

$$ds^2 = -c^2 dt^2 + (d\mathbf{r} - \mathbf{v}_s dt)^2 \longleftarrow ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

reference frame for phonon is dragged
by moving liquid

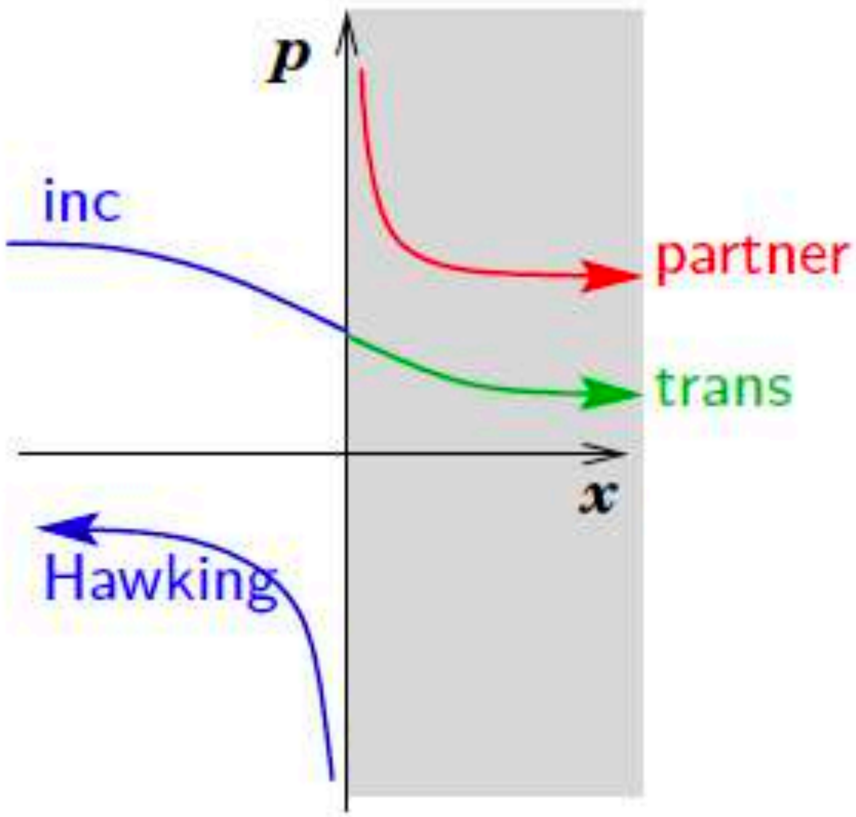
geometry is emergent

G. E. Volovik, Simulation of Panleve-Gullstrand black hole in thin 3He-A film,
Pis'ma ZhETF, 69, p. 662-668 or JETP Lett., 69, p. 705-713 (1999).

Tunneling Derivation of Hawking Temperature

long wave length limit : $E - v(x) \cdot p = \pm c(x) \cdot p \quad \rightarrow \quad p = \dfrac{E}{v(x) \pm c(x)}$

phase space :



Tunnel probability

$P \propto e^{-2S/\hbar}$ where $S = \left| Im \int p(x)\, dx \right|$

near the horizon

$$\frac{E}{v(x) - c(x)} \simeq \frac{E}{(x \pm i\epsilon)\, \frac{d}{dx}(v-c)\big|_0} \rightarrow \pm i\delta(x) \frac{E}{\frac{d}{dx}(v-c)\big|_0}$$

$$S \simeq \frac{\pi E}{\frac{d}{dx}(v-c)\big|_0}$$

Hawking temperature

$P \propto e^{-E/(k_B T_H)}$ with $k_B T_H = \dfrac{\hbar}{2\pi} \left| \dfrac{d}{dx}(v-c) \right|_0$

Surface Gravity: $\kappa = \left(\dfrac{d(v \pm c)}{dx} \right)_{x_H}$

Courtesy Nicolas Pavloff

$$k_B T_H = \frac{\hbar}{2\pi} \kappa$$

Gravity wave analogues of black holes

Ralf Schützhold* and William G. Unruh†
Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z1
(Received 22 May 2002; published 28 August 2002)

It is demonstrated that gravity waves of a flowing fluid in a shallow basin can be used to simulate phenomena around black holes in the laboratory. Since the speed of the gravity waves as well as their high-wave-number dispersion (subluminal vs superluminal) can be adjusted easily by varying the height of the fluid (and its surface tension) this scenario has certain advantages over the sonic and dielectric black hole analogs, for example, although its use in testing quantum effects is dubious. It can be used to investigate the various classical instabilities associated with black (and white) holes experimentally, including positive and negative norm mode mixing at horizons.

« For the Einsteinians, the ds^2 has a mystical and universal significance, constraining all phenomena to fit themselves in the mold of a sort of space-time form, like water in a vase. »

Paul Painlevé.

$$\left(\frac{\partial}{\partial t}+\boldsymbol{v}_{\mathrm{B}}^{\parallel}\cdot\boldsymbol{\nabla}_{\parallel}\right)^2\delta\Phi_{(0)}-g\,h_{\mathrm{B}}\boldsymbol{\nabla}_{\parallel}^2\,\delta\Phi_{(0)}=0$$

$$kh << 1$$

$$\Box\,\delta\Phi_{(0)}=\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\;g^{\mu\nu}\partial_\nu\,\delta\Phi_{(0)})=0$$

$$\mathrm{g}_{\mathrm{eff}}^{\mu\nu}=\begin{pmatrix}1 & \boldsymbol{v}_{\mathrm{B}}^{\parallel}\\ \boldsymbol{v}_{\mathrm{B}}^{\parallel} & \boldsymbol{v}_{\mathrm{B}}^{\parallel}\otimes\boldsymbol{v}_{\mathrm{B}}^{\parallel}-g\,h_{\mathrm{B}}\mathbf{1}\end{pmatrix}$$

$$c=\sqrt{gh}$$

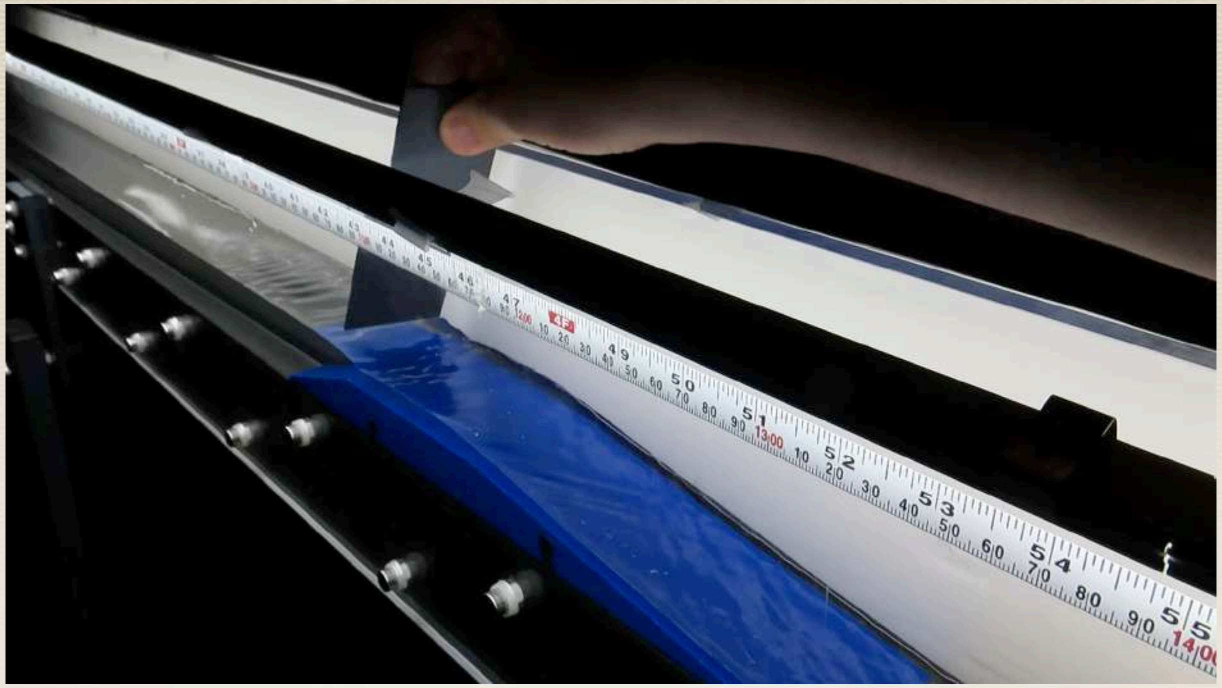
$$ds^2_{P.-G.}=c^2dt^2-(d\vec{x}-\vec{U}dt)^2$$

$$U(x) <=> V_{Schw}(r)=\sqrt{\frac{2GM}{r}}$$

PHYSICAL REVIEW D **66**, 044019 (2002)

Analogue Black Hole Flows

By courtesy of Eric Lamballais for the lending of the pedagogical open channel

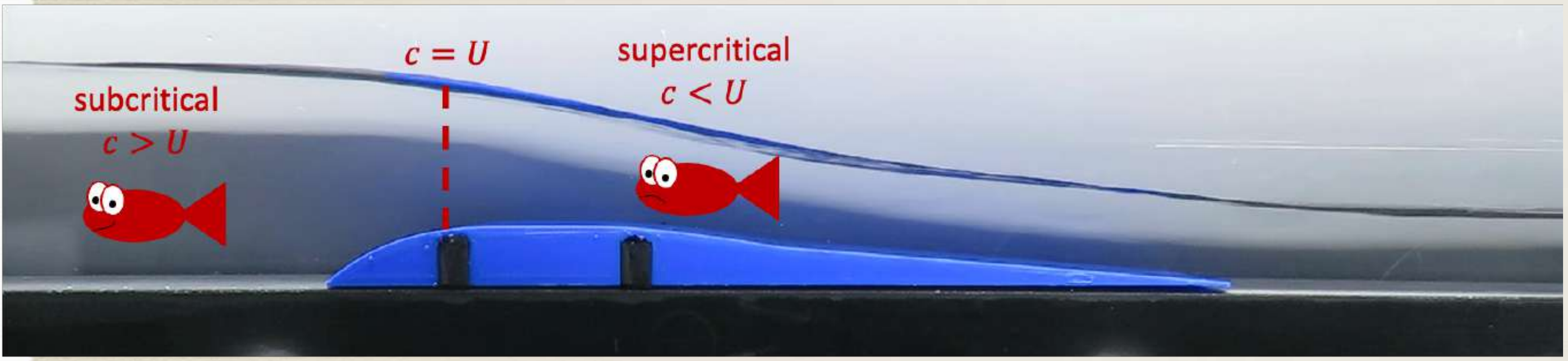


There is NO initial water depth !

$$Fr = \frac{U}{c} = \frac{U}{\sqrt{gh}}$$

when

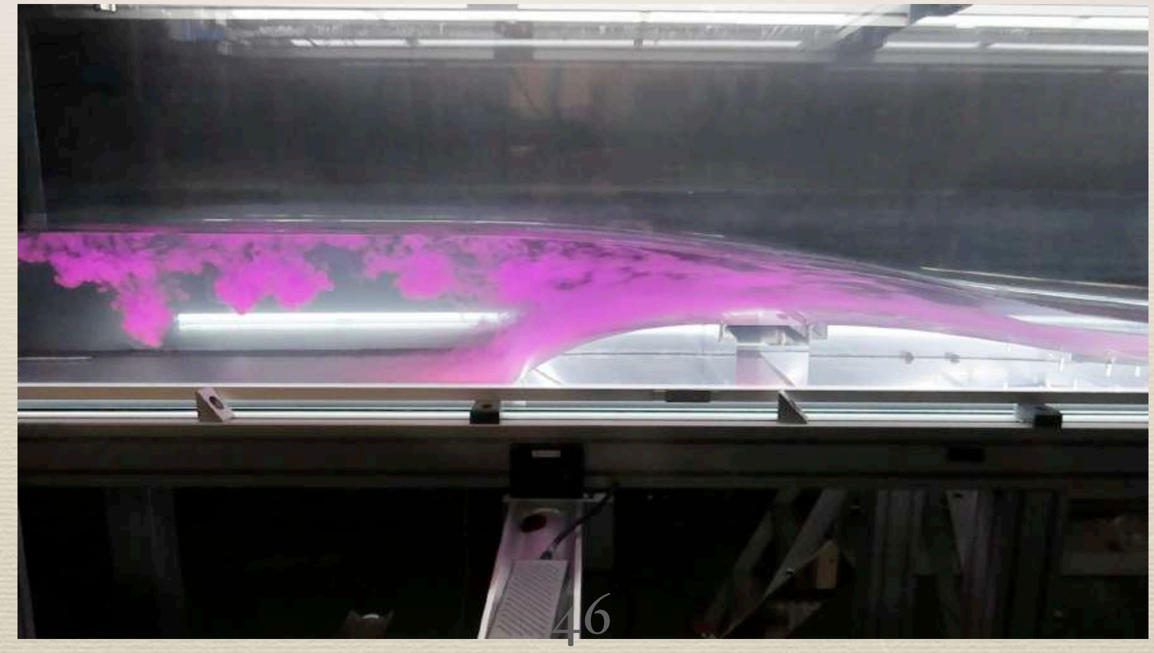
$$kh << 1$$



Small open channel features:

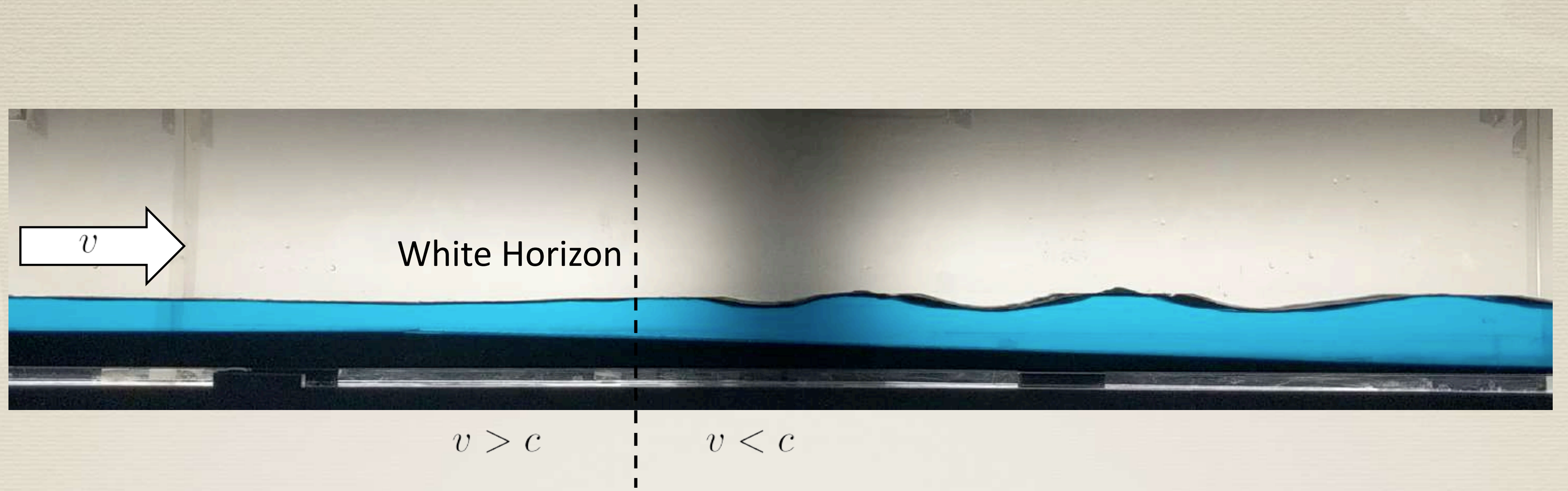
- Length : L=2,5 m
- Width : w=5,4 cm
- Flow rate range in L/min: 2-38
- Flow rate range in m^2/s: 0.0006-0.0115

Alexis Bossard,
PhD Thesis to be defended in december 2025
Medium Water Channel of the Pprime Institute (France)



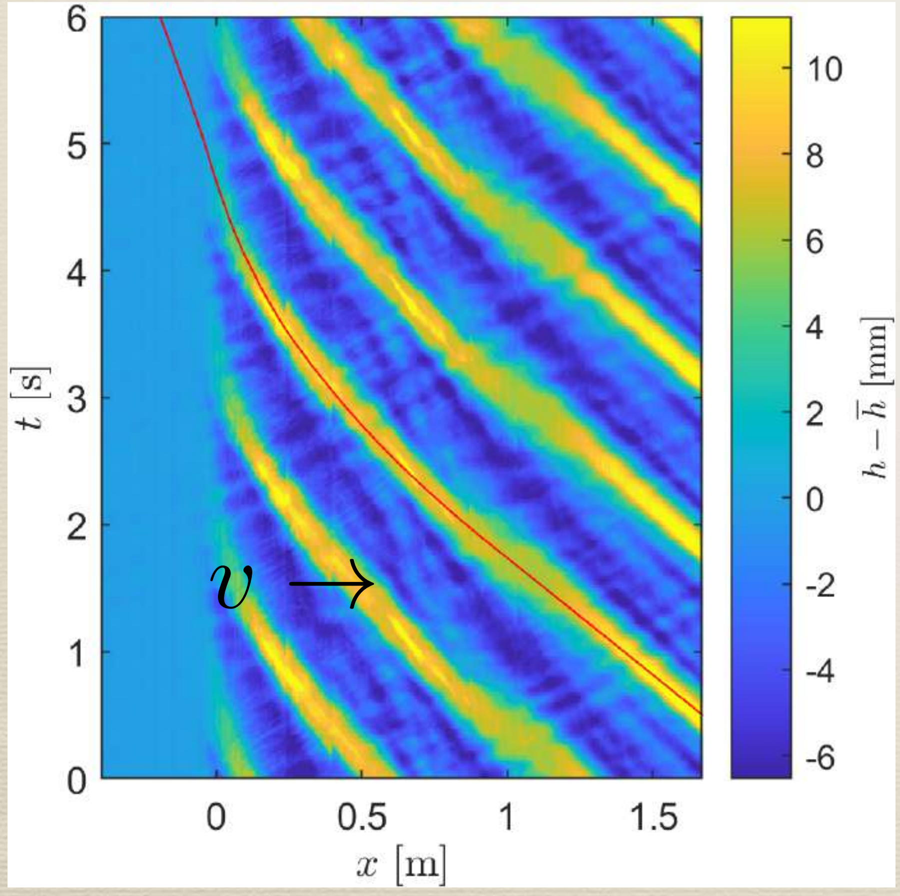
There is an initial water depth !

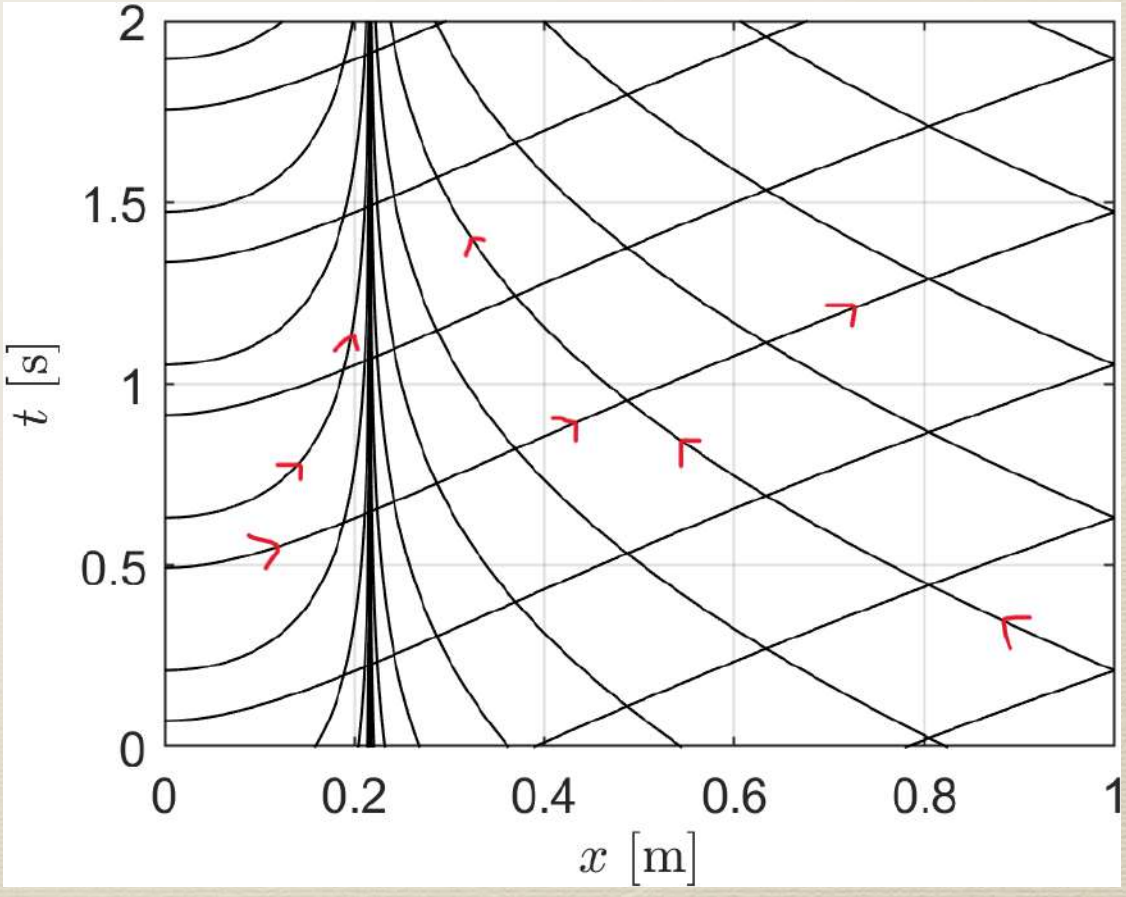
Space-Time Diagrams of a White Fountain=Cataract



Water interface recording

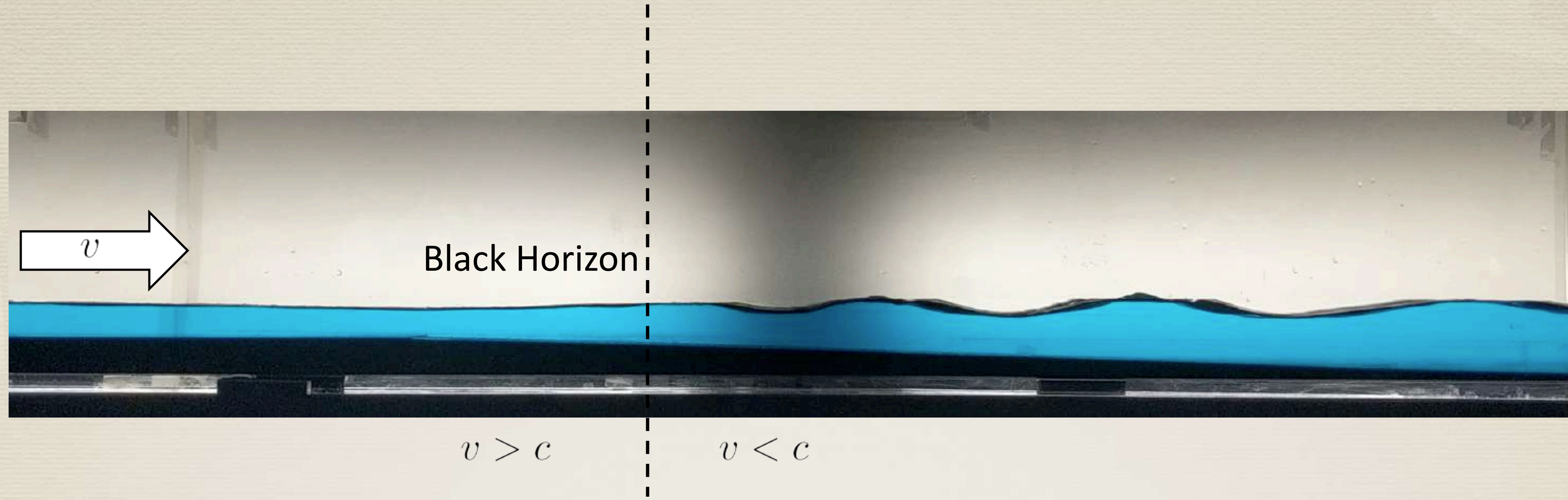
Spacetime diagram





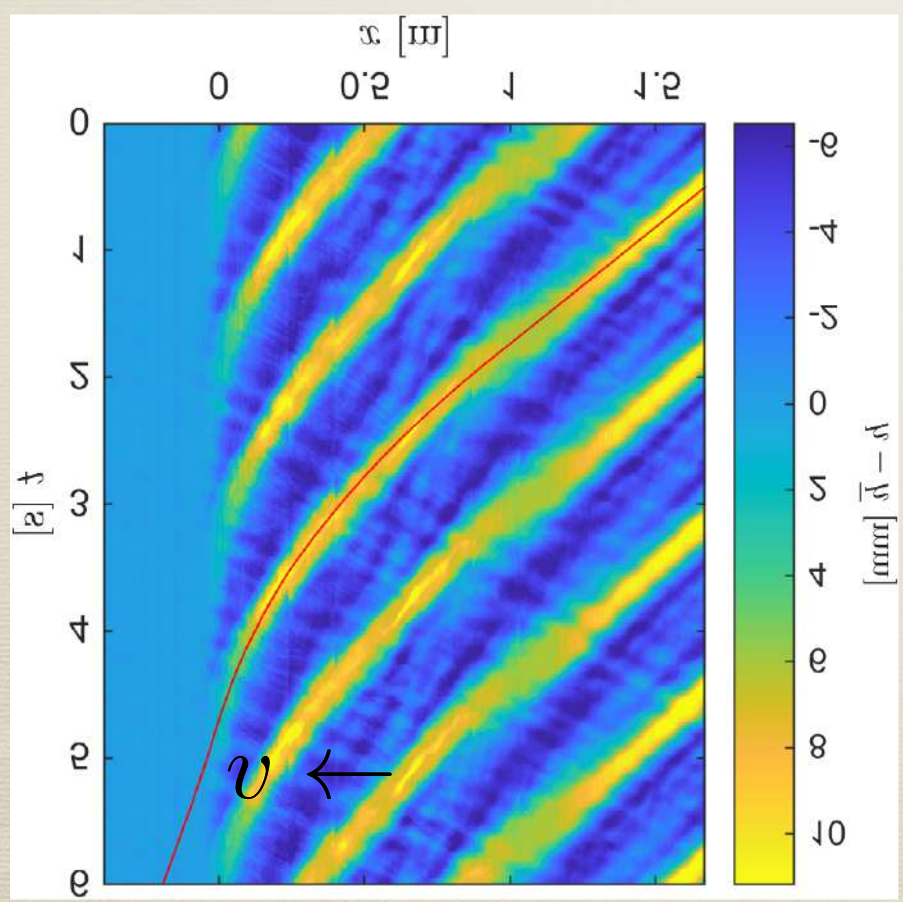
Courtesy Jaro Fransen from his Master Thesis at Eindhoven University of Technology (2024).

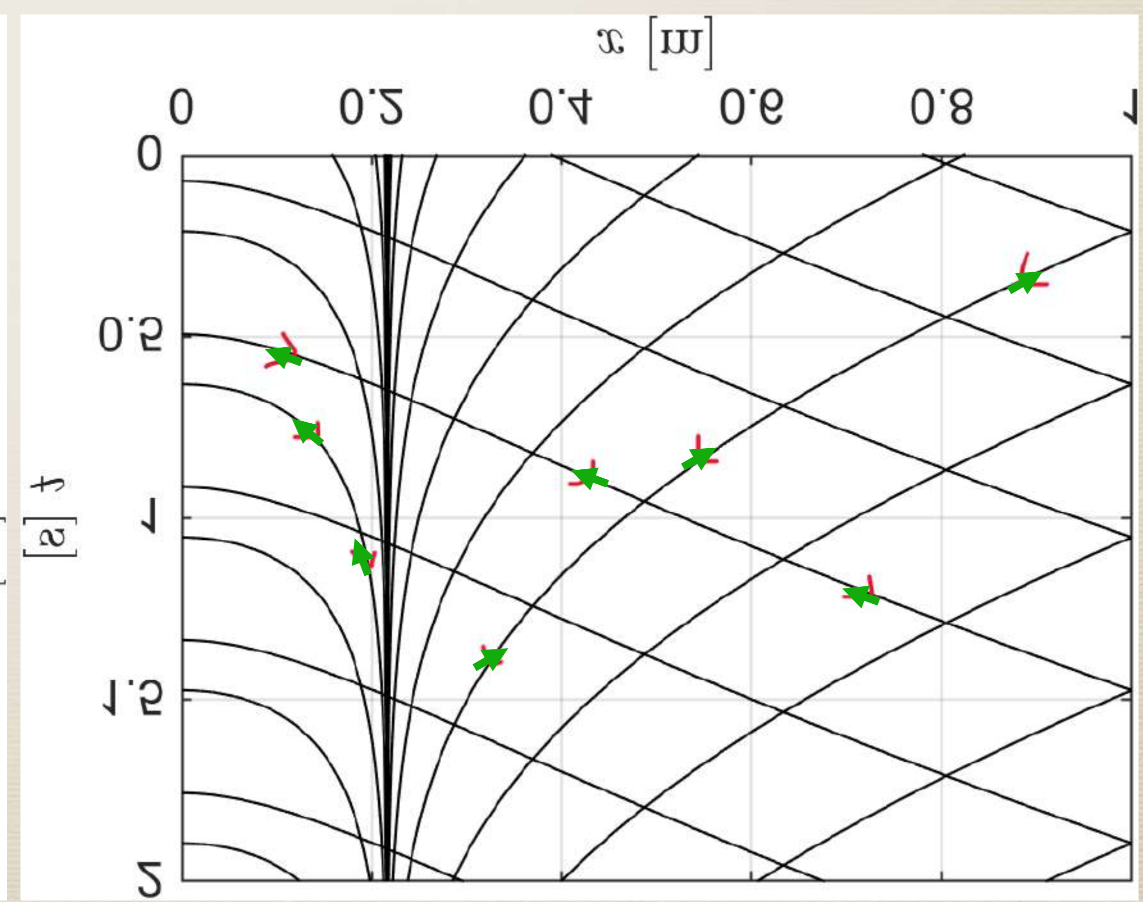
Space-Time Diagrams of a Black Hole=Waterfall



Water interface recording

Spacetime diagram



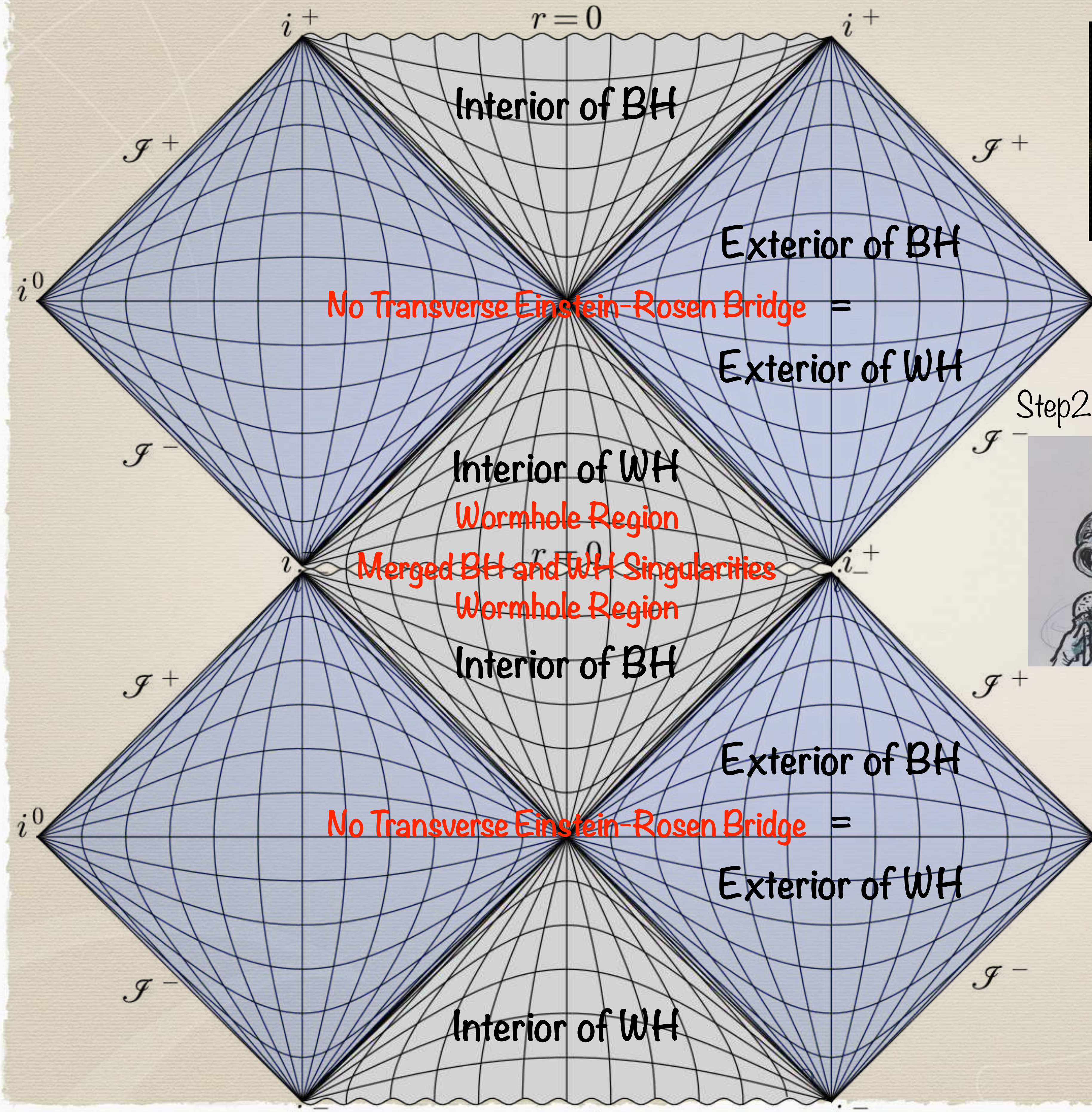


After Jaro Fransen from his Master Thesis at Eindhoven University of Technology (2024).

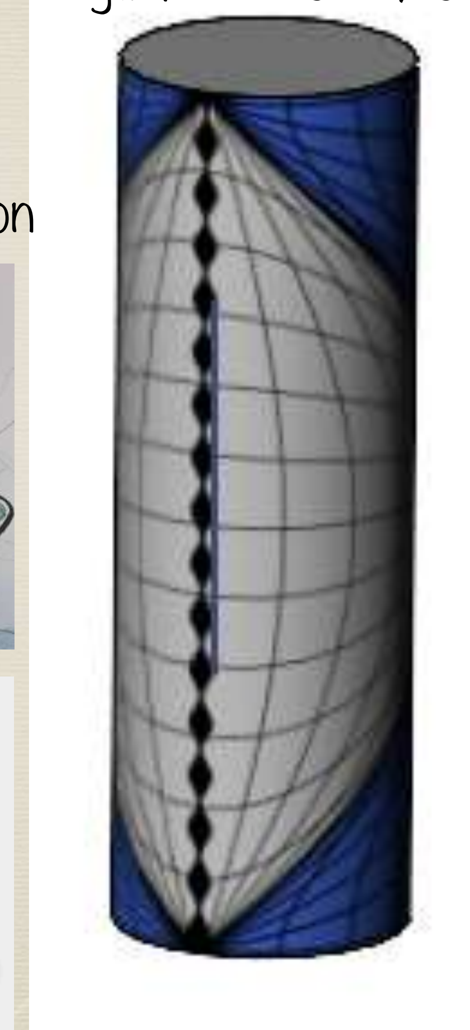
An Aquatic Wormhole with its Waterfall and related Cataract in Luxembourg

Pulling Pond (Schiessentümpel) of the Black Ernz River (Schwaarz Iernz)

Folding the Penrose Diagram to Connect Past and Future Singularities of a NR-BH/WH pair

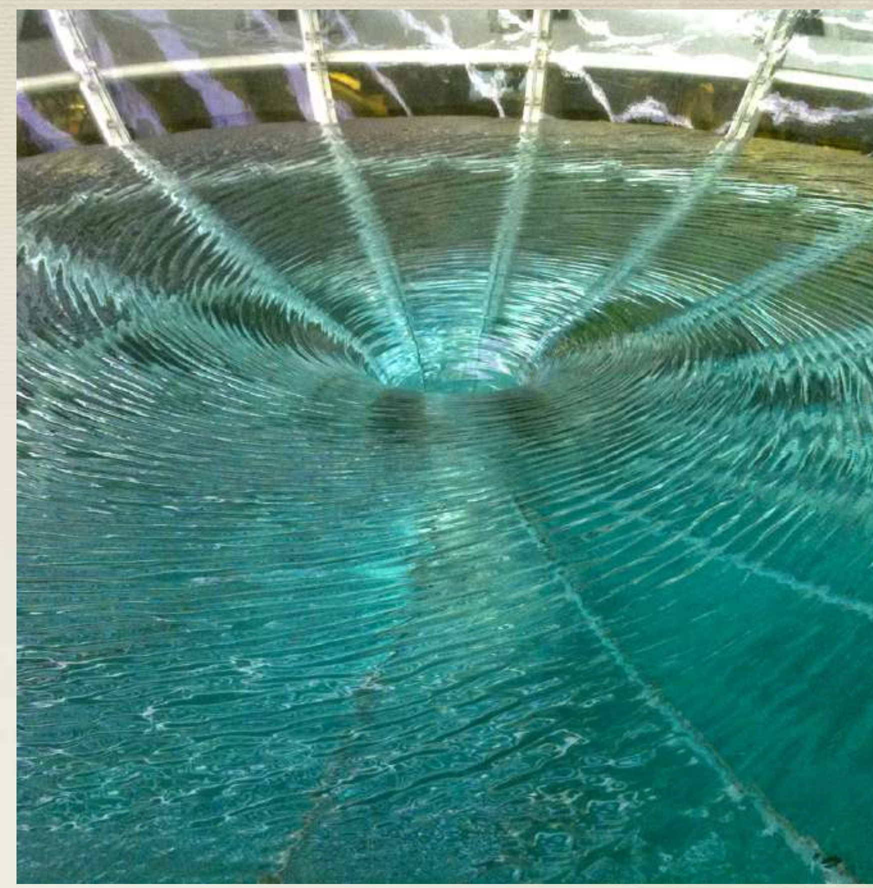


Step 1: singularities connexion



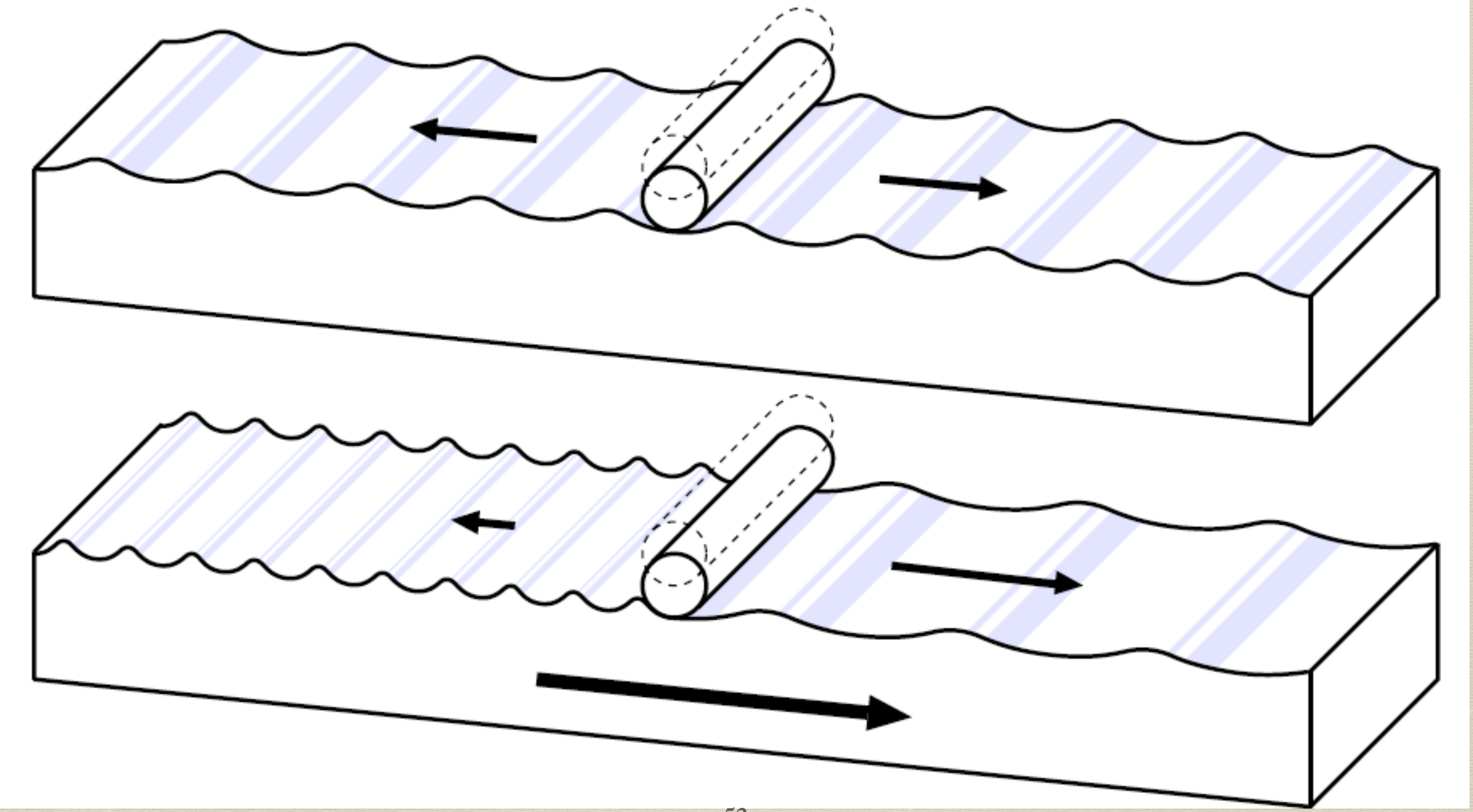
Step2: exteriors connexion

Experimentally Connected Past and Future Singularities of a R-BH



https://nedkahn.com/portfolio/rain-oculus

The Doppler Effect :
Blue-Shifting versus Red-Shifting



Intriguing Consequences of the Doppler Effect

$$kh << 1 \quad (\omega - \mathbf{U.k})^2 \simeq c^2k^2 \quad c = \sqrt{gh}$$

+ For a dispersion relation with an "acoustic" branch,

the wavelength goes to zero at the blocking line

$$\omega = \Omega(k) = \mathbf{U.k} \pm c|k|$$

$$k_I = \frac{\omega}{U + c} \quad U < 0$$

$$U \rightarrow -c \qquad \lambda_I \rightarrow 0$$

=> UV Problem at Blocking

+ Negative relative frequencies could appear

$$\omega' = \omega - \mathbf{k.U} < 0$$

=> Theoretical Curiosity ?

Invariants of the Massless Scalar Field Equation

$$ds^2 = [c^2 - U(x)^2]dt^2 + 2U(x)dt\,dx - dx^2$$

$$S = \frac{1}{2}\int dt\,dx\left[\frac{1}{c^2}\left|\partial_t\phi + U\partial_x\phi\right|^2 - \left|\partial_x\phi\right|^2\right]$$

$$\partial_t(\partial_t\phi + U\partial_x\phi) + \partial_x(U\partial_t\phi + U^2\partial_x\phi) - c^2\partial_x^2\phi = 0$$

$$\phi \to e^{i\alpha}\phi$$

$$N = \frac{i}{2c^2}\int_{-\infty}^{\infty} dx\,[\phi^*(\partial_t\phi + U\partial_x\phi) - \phi(\partial_t\phi^* + U\partial_x\phi^*)]$$

$$t \to t + a$$

$$E = \frac{1}{2}\int_{-\infty}^{\infty} dx\left[\frac{1}{c^2}|\partial_t\phi|^2 + (1 - U^2/c^2)|\partial_x\phi|^2\right]$$

$$U = cste$$

$$N = \frac{1}{c^2}\int_{-\infty}^{\infty} dk(\omega - Uk)|\tilde{\phi}(k)|^2$$

$$E = \frac{1}{c^2}\int_{-\infty}^{\infty} dk\,\omega(\omega - Uk)|\tilde{\phi}(k)|^2$$

The Wave Energy for Gravity Waves

In absence of a current :

$$E_0 = \frac{\rho g}{2} A^2 = \frac{\rho \omega^2}{2k \tanh(kh)} A^2$$

In presence of a current :

$$E_U = \frac{\rho \omega (\omega - \mathbf{U}.\mathbf{k})}{2k \tanh(kh)} A^2$$

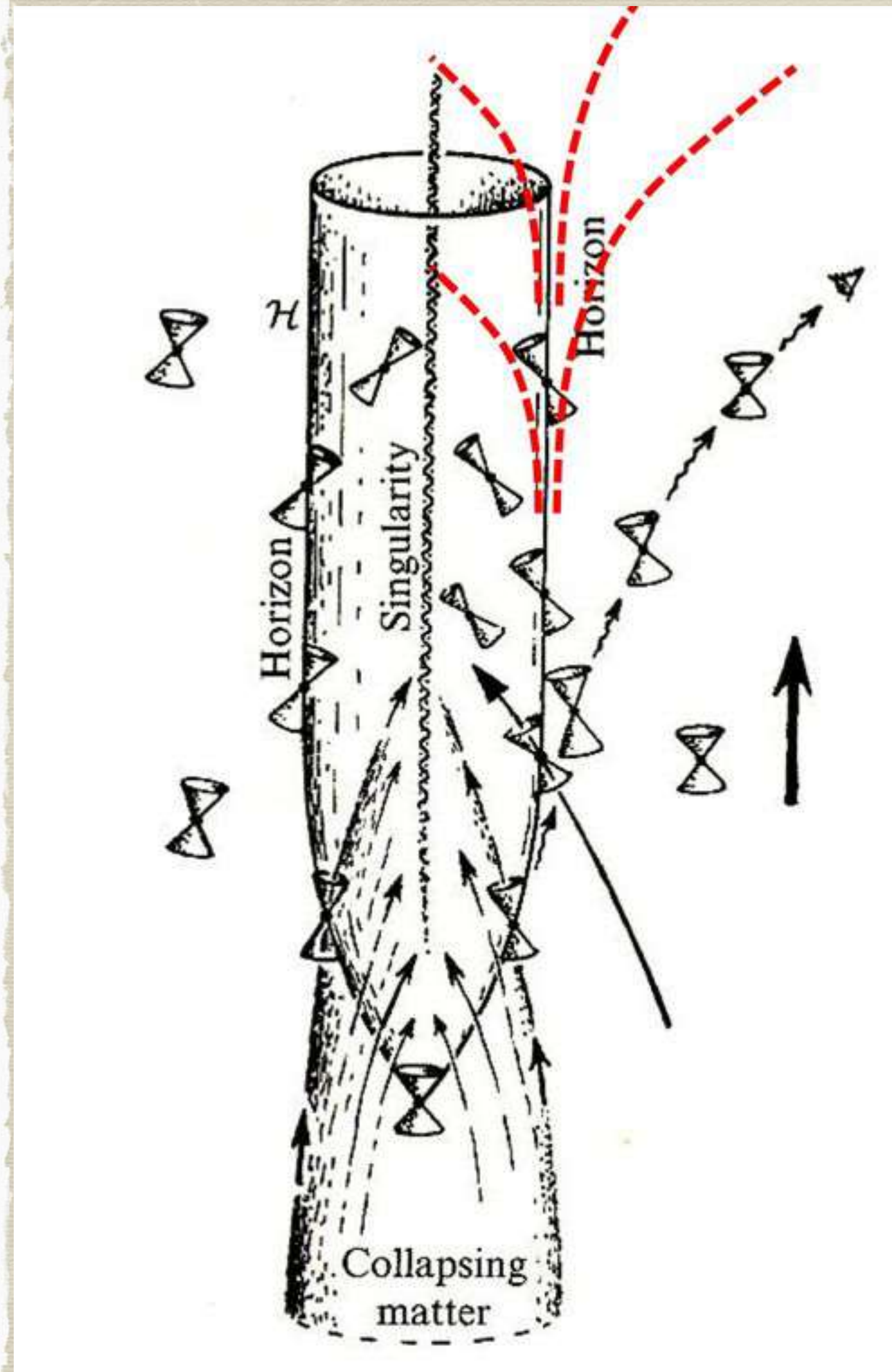
Invariance of Wave Action (number of particles in QM!) :

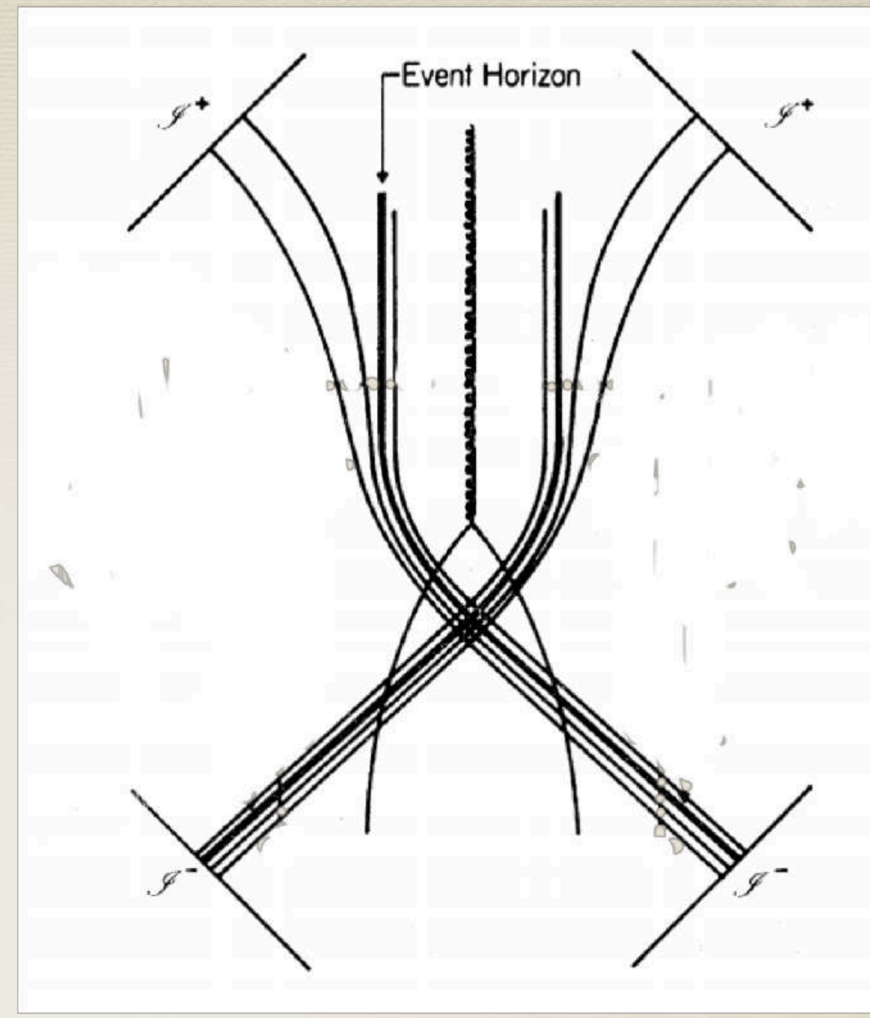
$$\frac{E_U}{\omega - \mathbf{U}.\mathbf{k}} = \frac{E_0}{\omega}$$

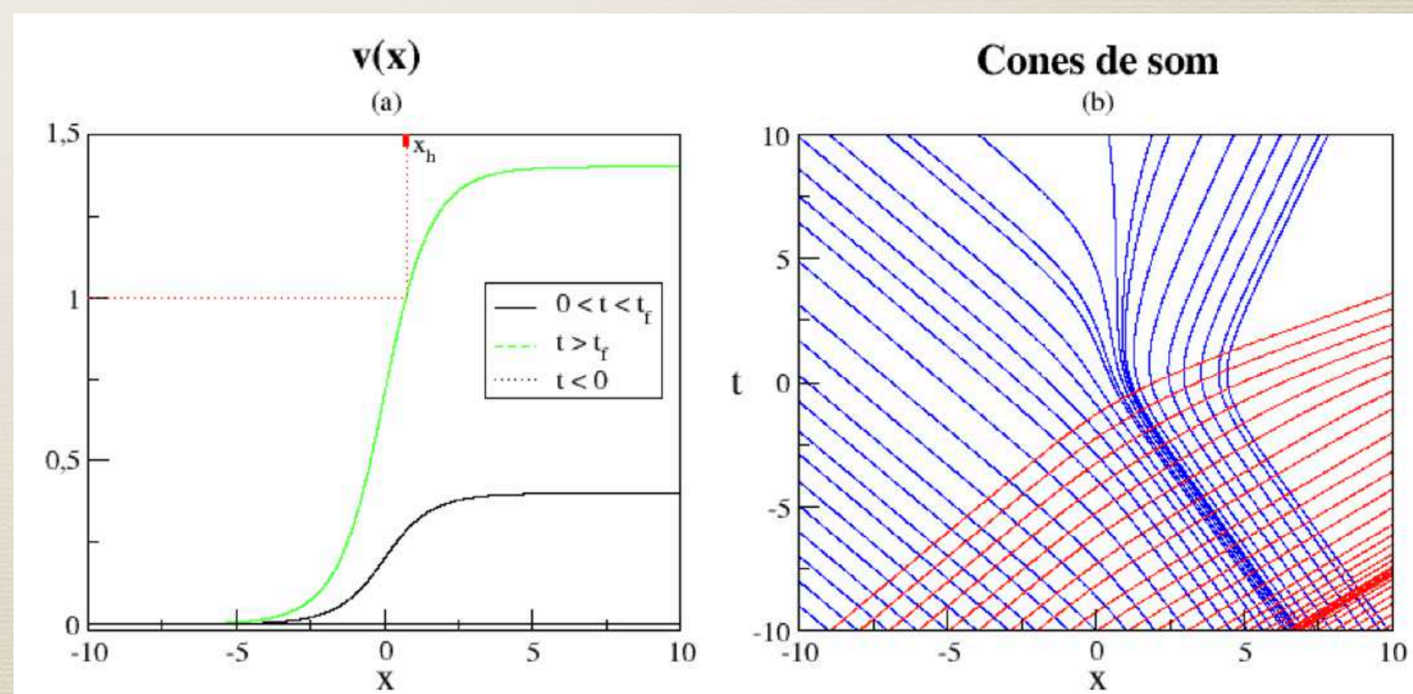
$$\omega - \mathbf{U}.\mathbf{k} = \pm \sqrt{gk \tanh(kh)}$$

Negative energy waves are waves with negative relative frequencies !

Ancestors of Hawking Radiation were created before the Collapse



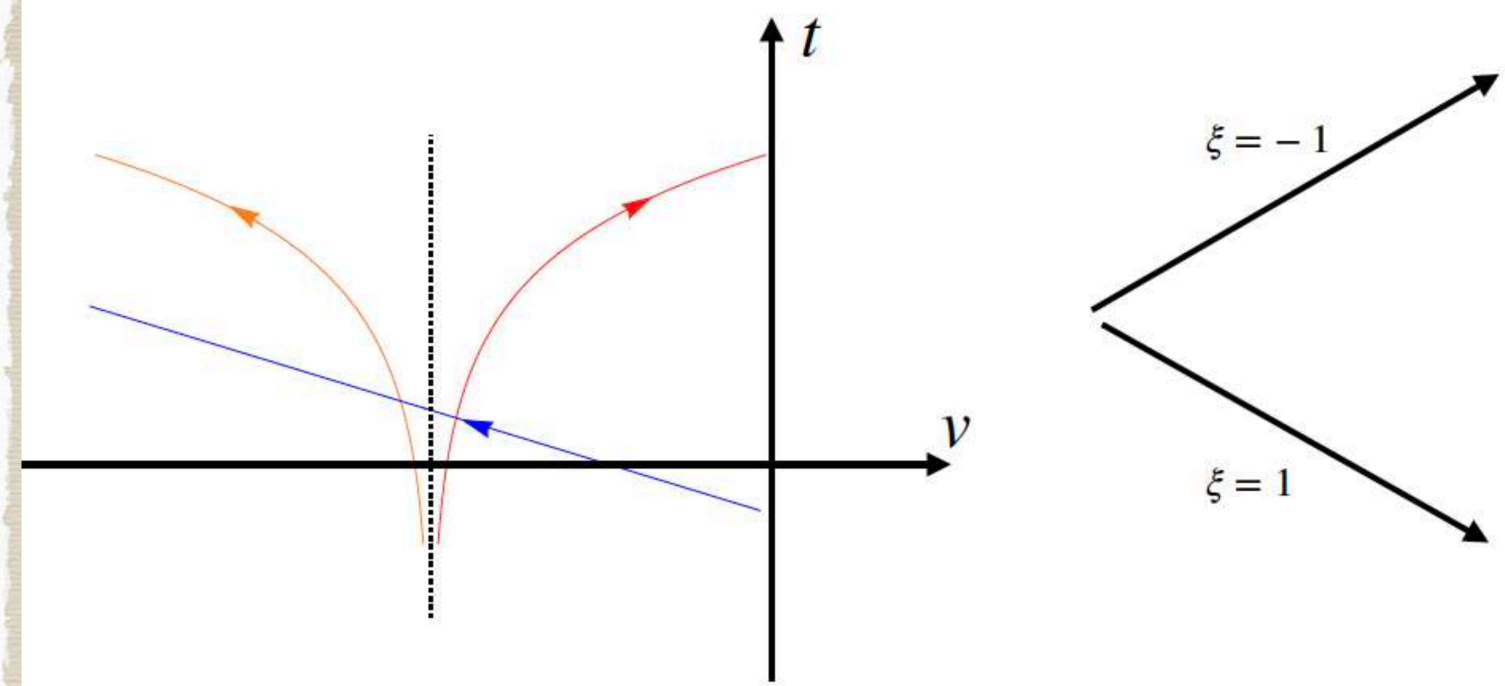


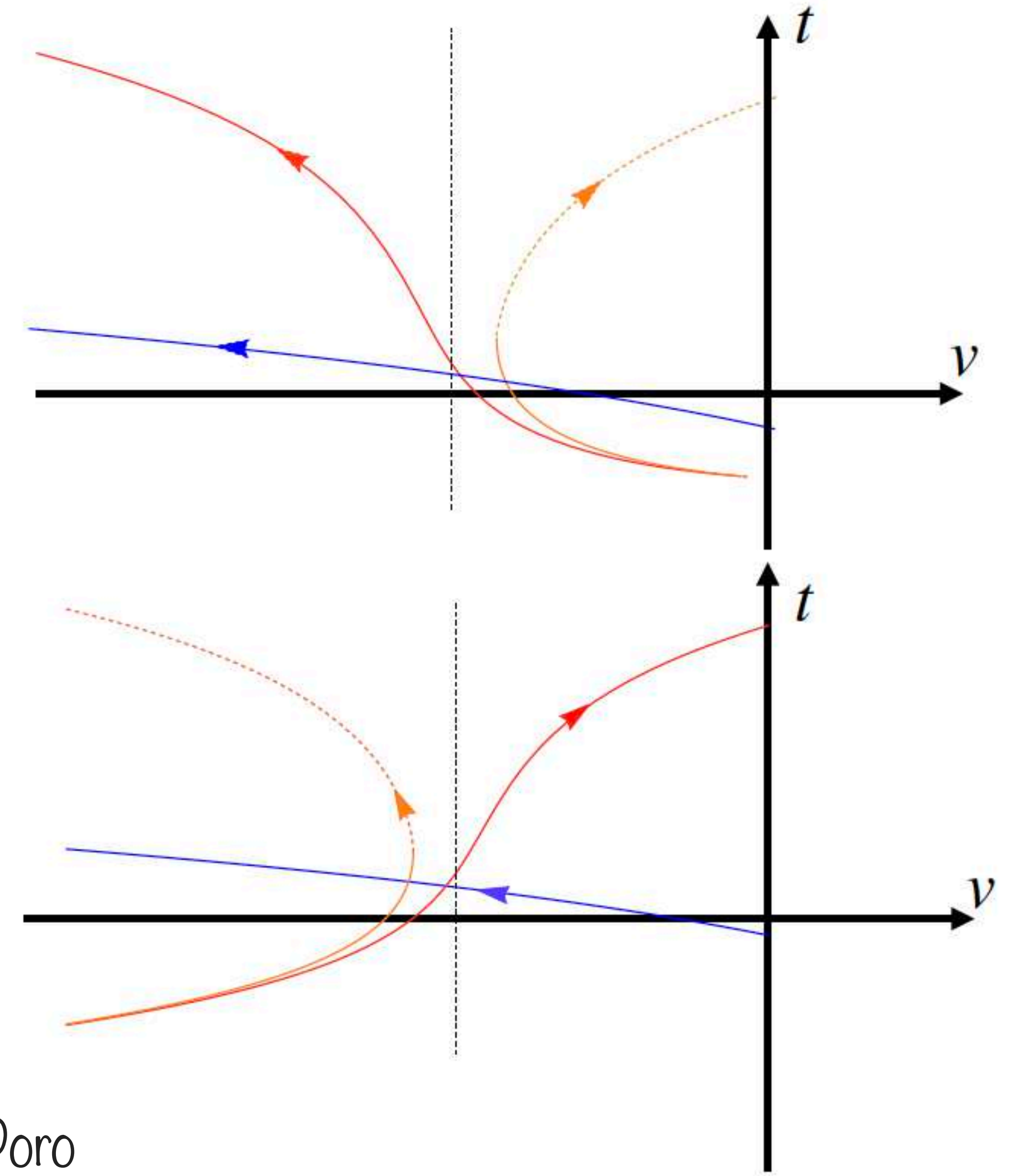


C. A. S. Maia, Análogos gravitacionais em matéria condensada, Tese (Doutorado em Física Teórica), Universidade Estadual Paulista, São Paulo, 2008

Hawking Radiation With Modified Dispersion Relation

$$\omega^2 = c_s^2 k^2 \longrightarrow \omega^2 = c_s^2 k^2 \left(1 + \xi \frac{c_s^2 k^2}{\Lambda^2} \right)$$





Hawking radiation is supposed to be robust against

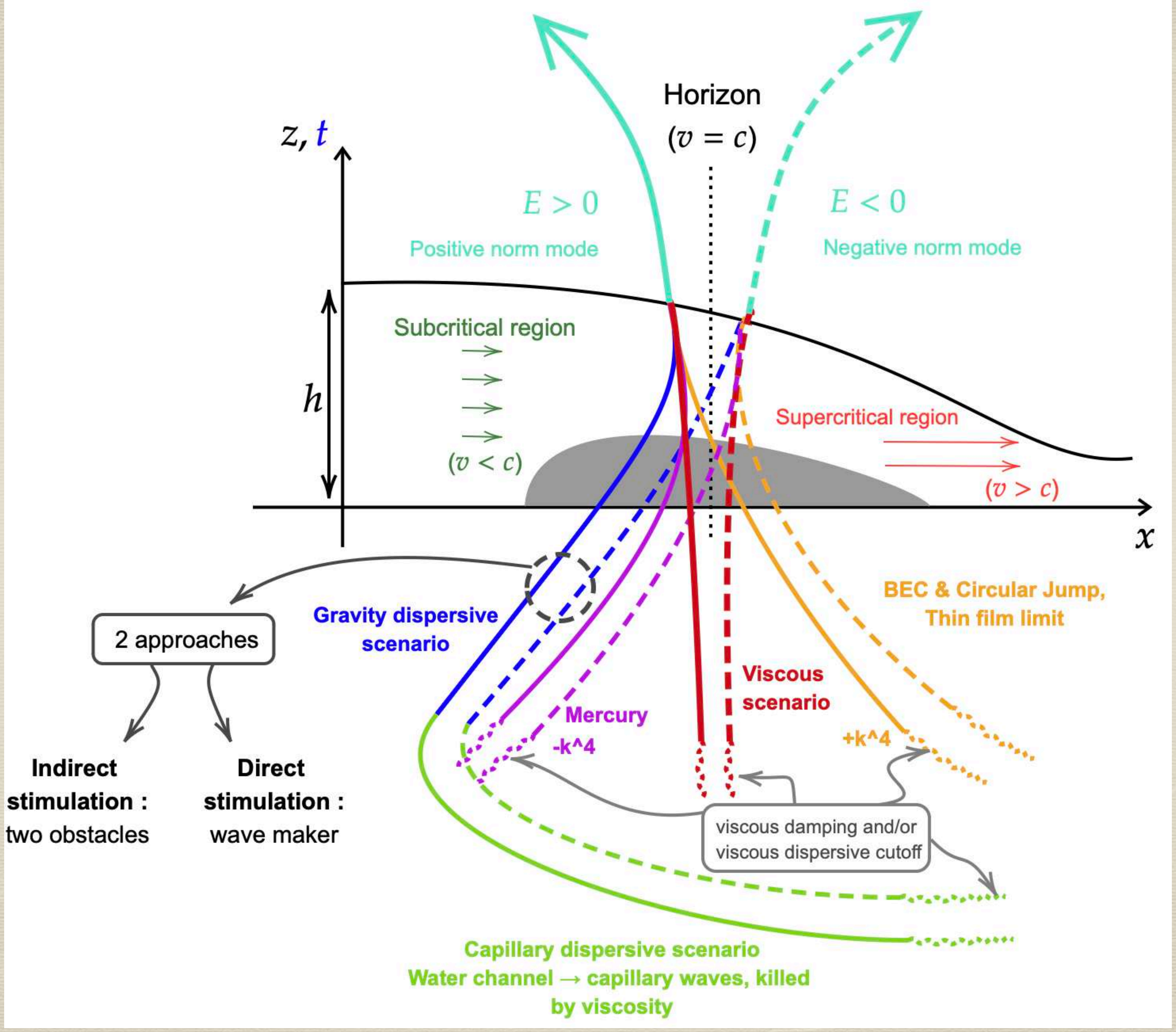
the modification of the dispersion relations

Courtesy Francesco del Poro

Francesco del Poro, Beyond Lorentz invariance: a journey from Analogue to Hořava Gravity,
PhD Thesis (2024) under the supervision of Stefano Liberati
https://iris.sissa.it/handle/20.500.11767/141091

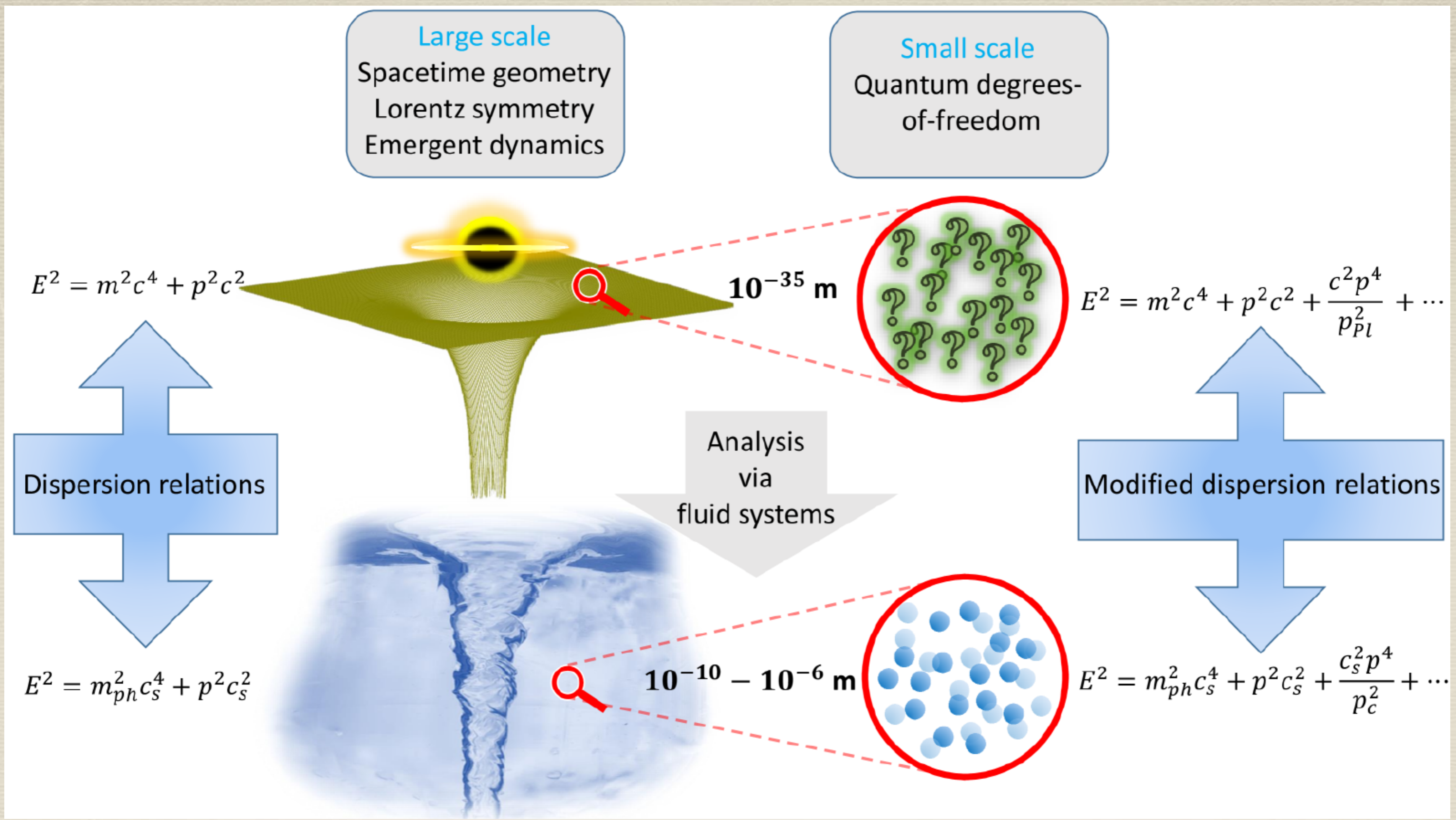
Francesco Del Porro, Stefano Liberati and Marc Schneider,Tunneling method for Hawking quanta in analogue gravity,
Comptes Rendus Physique 25, S2, p. 1-27 (2025)

Many Dispersive and Dissipative Scenarii for Wave Scattering at a BHH



Courtesy Ludivine Goncalves (current PhD Thesis)

Analogue Simulations of Quantum Gravity with Fluids



In fluids, the effective geometry breaks down at scales where the continuous description in terms of macroscopic variables no longer holds, in close analogy to the breakdown of spacetime geometry due to quantum gravitational effects. This scale is manifest by a breakdown of Lorentz symmetry as evidenced by the above modified phonon dispersion relations (see figure, right). There, m_{ph} is the mass of phonon excitations which is the analogue of the rest mass m, with energy and momentum being denoted by E and p for either system. For $m_{ph} = 0$ and truncating the expansion at the fourth-order in p the above expression reduces to the well-known Bogoliubov dispersion relation in a Bose gas [100, 101] or in a circular hydraulic jump [104, 105]. The critical momentum $p_c = Mc_s$, that here plays the role of the Planck momentum p_{Pl}, depends on the mass M of the particles forming the condensate and is inversely proportional to the coherence length $\xi = \hbar/(Mc_s)$. When $p \ll p_c$, the excitations follow the standard relativistic energy-momentum relation.

S. L. Braunstein, M. Faizal, L. M. Krauss, F. Marino, and N.A. Shah, Analogue simulations of quantum gravity with fluids, Nature Reviews Physics, 5 (10), p. 612-622 (2023).