Quantum Forces

Emergent Effects of the Dynamic Vacuum

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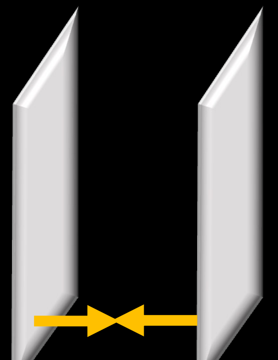
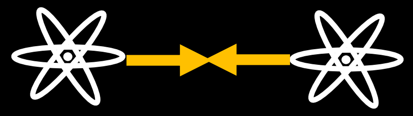
Dispersion Forces

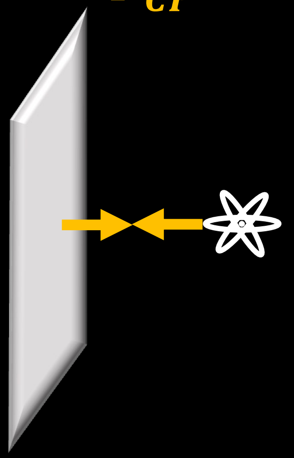
- Driven by quantum fluctuations
  - Casimir
  - Van der Waals (vdW) & Casimir-Polder (CP)



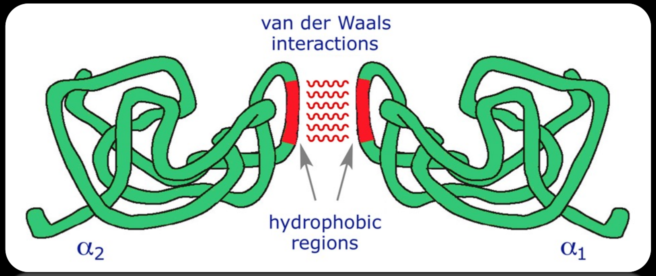
$F_{Casimir}$



F_{vdW} & F_{CP}

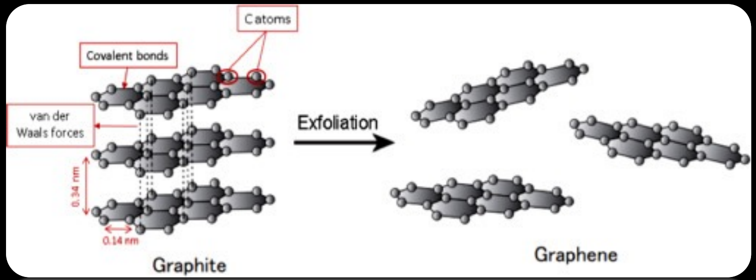


- Structure, stability, and function for molecules and materials
  - Proteins



Source: www.stereoelectronics.org
vdW attractions between non-polar amino acid side chains contribute to protein stability

  - Nanostructures



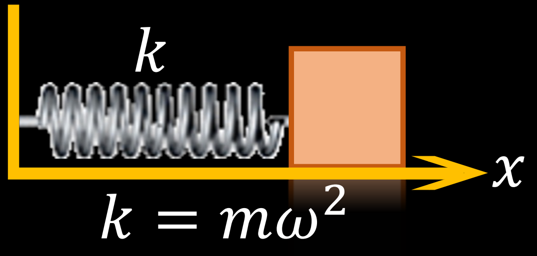
Source: https://doi.org/10.1016/j.actbio.2021.06.047

  - molecular solids
  - crystalline surfaces . . .

Zero-point Energy

- Classical Harmonic Oscillator

$$\begin{cases} m\ddot{x} + kx = 0 \quad , \quad p = m\dot{x} \\ E = \dfrac{p^2}{2m} + \dfrac{1}{2} m\omega^2 x^2 \end{cases}$$



  - $x\,,p$ classical variables
  - $E_{\min} = 0$

$$[x,p] = i\hbar \;\;, p = -i\hbar \frac{d}{dx}$$

QM

- Quantum Harmonic Oscillator

$$\begin{cases} H\,\psi(x) = E\,\psi(x) \quad , \quad H = \dfrac{p^2}{2m} + \dfrac{1}{2} m\omega^2 x^2 \\ E = \hbar\omega\left(n + \dfrac{1}{2}\right) \;\;, \;\; n = 0, 1, 2, \ldots \end{cases}$$

  - $x\,,p$ QM operators
  - $E_{\min} = \frac{1}{2}\hbar\omega$

Zero-point Energy: Electromagnetic Field

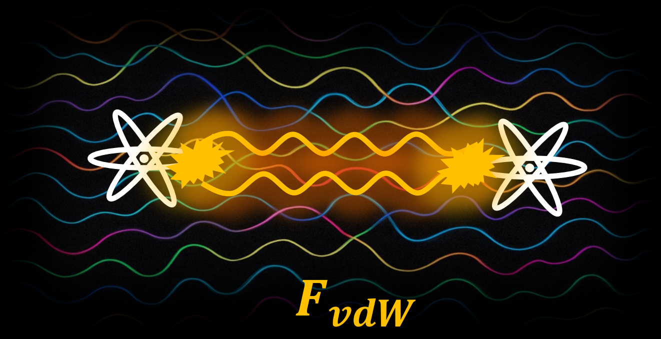
- Maxwell equations (potentials) $\longrightarrow$ wave equations
- Single-mode equation: $\ddot{\boldsymbol{\mathcal{A}}}_k + c^2 k^2 \boldsymbol{\mathcal{A}}_\mathrm{k} = 0$
- Each mode of EM field $\longrightarrow$ harmonic oscillator
  - $E_{\min}(k) = 0$

QM

- Quantum Electromagnetic Field
  - $E_{\min}(k) = \frac{1}{2}\hbar\omega_k$
  - Infinite modes $\longrightarrow$ infinite zero-point EM energy
  - Excitation of EM oscillators $\longrightarrow$ photon

C. Cohen-Tannoudji et al, "Photons and Atoms: Introduction to Quantum Electrodynamics", John Wiley & Sons, 1997

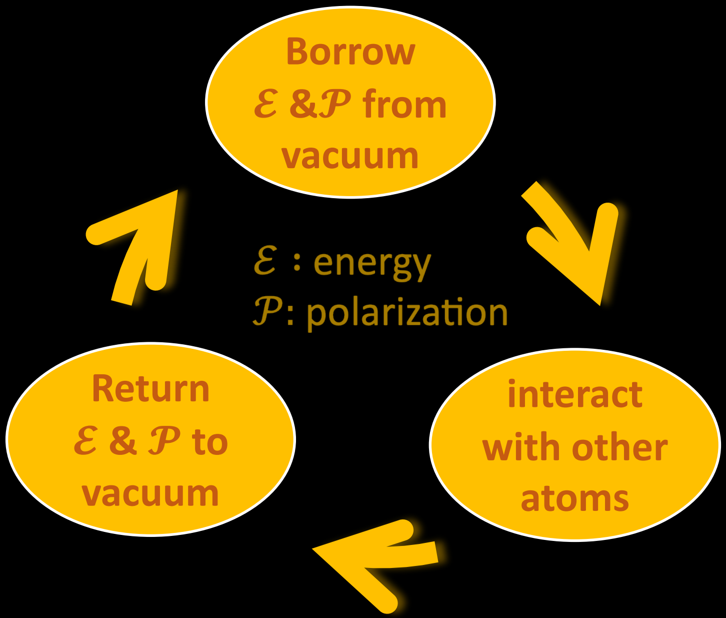
vdW Forces



- Mediated by vacuum EM field → exchange (virtual) photons
- Photons propagate at finite speed → retardation effects
  - Short distance: non-retarded London dispersion $(\Delta E \propto -R_{ab}^{-6})$
  - Long distance: retarded Casimir-Polder force $(\Delta E \propto -R_{ab}^{-7})$

    H. B. G. Casimir, D. Polder, Phys. Rev. 73 (1948)

- Borrowed energy subject to energy-time uncertainty
  - small R_{ab} → high-frequency modes contributions
  - large R_{ab} → low-frequency modes contributions



- **Excited atoms:** borrow/lend $\mathcal{E}$ & $\mathcal{P}$ → $\pm$force (resonant and off-resonant)

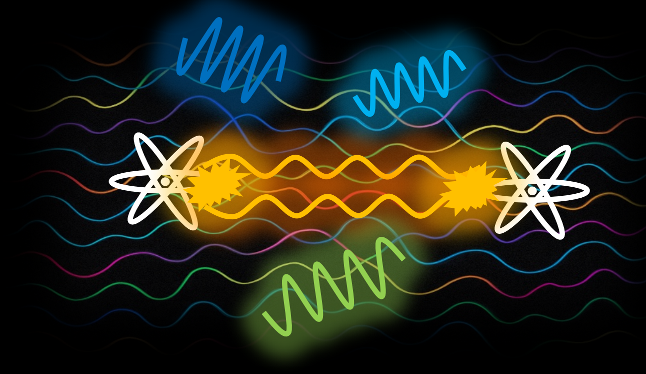
  L. Gomberoff, R. R. McLone, E. A. Power, J. Chem. Phys. 44 (1966)
  H. Safari, R. Karimpour, PRL 114 (2015)

vdW Forces: External Fields & Boundaries

- **External fields**
  - Dynamic Fields ⟶ excitation of vacuum EM field
    - Near zone: $\Delta E \propto -R_{ab}^{-1}$
    - Far zone: $\Delta E \propto \pm R_{ab}^{-2}$

      (pair-orientation averaged)

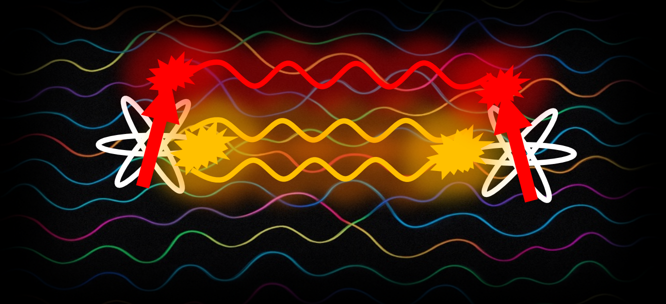
      T. Thirunamachandran, Molecular Physics 40 (1980)



  - Static Fields ⟶ $\Delta E_{es} \propto \pm R_{ab}^{-3}$ & $\Delta E_{pol} \propto -R_{ab}^{-6}$

    R. Karimpour, D. Fedorov, A Tkatchenko, PRR 4 (2022)

    R. Karimpour, D. Fedorov, A Tkatchenko, PRL 121 (2022)

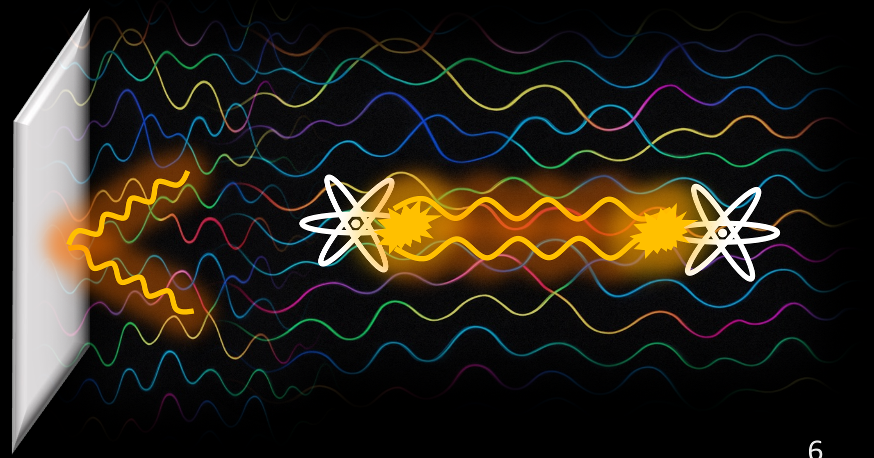


- **Boundaries**
  - Scattered modes from boundaries

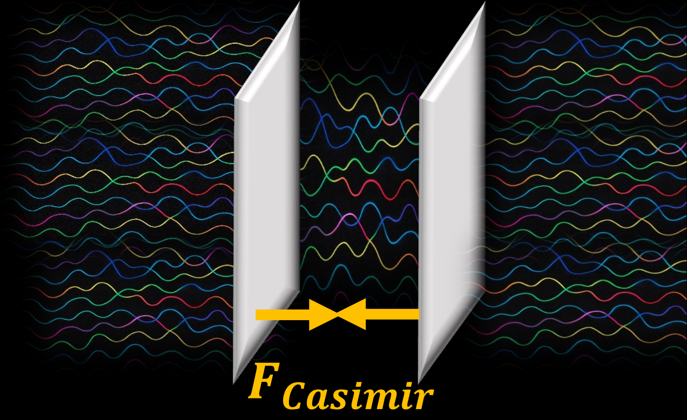
$$Field = Field^{(0)} + Field^{(S)}$$

Macroscopic QED ✅

S. Y. Buhmann, "Dispersion Forces I & II", Springer, 2013



Casimir Force



- Boundaries reshape fluctuations
  - Outside the plates → mode spectrum remains full
  - Between the plates → some modes fit between the plates
  - Imbalance between inside & outside fluctuations → net pressure
  - Casimir (1948): attraction between two perfectly conducting plates ($\frac{F}{A} = -\frac{\pi^2 \hbar c}{240\, d^4}$)

    H.B.G. Casimir, Proc. Kon. Nederland. Akad. Wetensch 51, 793 (1948)

- Lifshitz Theory: real materials, finite temperature
  - Based on Rytov's "fluctuational electrodynamics"
  - Force → attraction/repulsion depending on setup

    E. M. Lifshitz, M. Hamermesh, Perspectives in theoretical physics, Pergamon (1992), pp. 329-349.

$$\nabla\times\boldsymbol{E} = i\frac{\omega}{c}\boldsymbol{H}\,, \qquad \nabla\times\boldsymbol{H} = -i\frac{\omega}{c}[\epsilon(\omega)\boldsymbol{E} + \boldsymbol{K}]$$

$$\langle K_i(\boldsymbol{r},\omega)K_j^*(\boldsymbol{r}',\omega')\rangle \propto \delta(\boldsymbol{r}-\boldsymbol{r}')\delta(\omega-\omega')\, Im[\epsilon(\omega)]\,\hbar \coth\frac{\hbar\omega}{2k_B T}$$

Zero-Point Energy: Fermionic Fields

- QM Fluctuations of Fields $\longrightarrow$ zero-point energy

- Dirac equation (free fermion): $\begin{cases} i\hbar\partial_t\psi = \mathcal{H}_D\psi \\ \mathcal{H}_D = \beta mc^2 + c\boldsymbol{\alpha}\cdot\boldsymbol{p} \end{cases}$, $\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}$, $\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$

  - Solutions: u_p , v_p for $E_p = \pm\sqrt{m^2c^4 + p^2c^2}$ $\longrightarrow$ $\psi = \sum_p (c_p u_p + c_{\bar{p}} v_{\bar{p}})$, $\bar{p} \to (-E, -\boldsymbol{p}, -\boldsymbol{s})$

$\{c_p, c_q^+\} = \delta_{pq}, \ldots$

$c_{\bar{p}} = b_p^+$, $c_{\bar{p}}^+ = b_p$

2ndQuant

- $\begin{cases} \Psi = \sum_p (u_p c_p + v_{\bar{p}} b_p^+) \\ \Psi^\dagger = \sum_p (u_p^* c_p^+ + v_{\bar{p}}^* b_p) \end{cases}$ $\longrightarrow$ $H_D = E_0 + \sum_p E_p (c_p^+ c_p + b_p^+ b_p)$,

$$\boxed{E_0 = -\sum_p |E_p|}$$

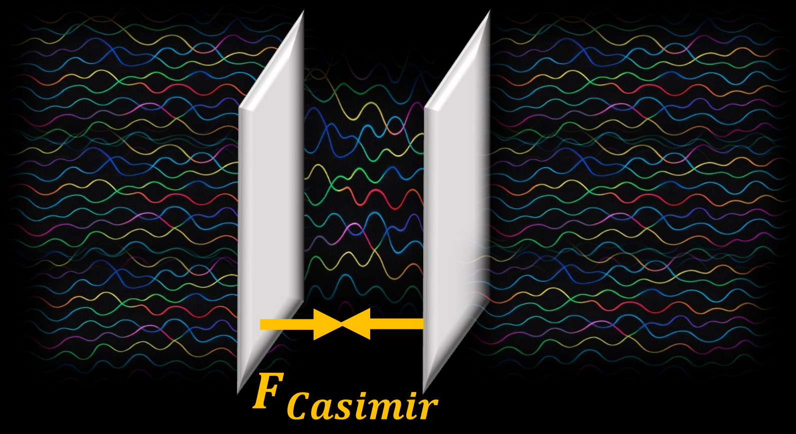
C. Cohen-Tannoudji et al, “Photons and Atoms: Introduction to Quantum Electrodynamics”, John Wiley & Sons, 1997

Fermionic Casimir Forces

- Two parallel plates restrict allowed momenta perpendicular to the plates
- Massless case (m=0) with bag boundary condition (no particle current through walls):

$$\frac{F}{A} = -\frac{7\pi^2 \hbar c}{960\, d^4} \qquad \left(F_D \to \frac{7}{4} F_{EM}\right)$$

- Boundary Condition $\longrightarrow$ Attractive/Repulsive Force



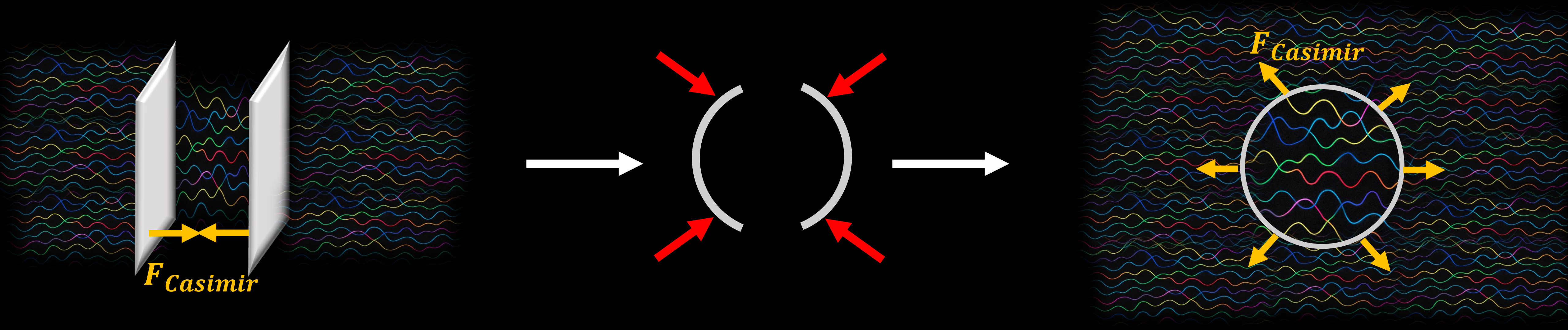
M. Bordag et al. "Advances in the Casimir effect", Vol. 145. OUP Oxford, 2009.

Challenges

- Infinite Energies
  - Mode sum for vacuum energy (with/without boundaries) $\longrightarrow$ infinite energy
  - Regularization & Renormalization
    - Math Techniques: e.g. Zeta function ($E = \sum \frac{\hbar\omega}{2} \rightarrow E(s) = \frac{\mu^{2s}}{2}\zeta_p\left(s - \frac{1}{2}\right)$ $\rightarrow$ identify divergencies $\rightarrow$ regularize), cutoffs, etc.
    - Green function approach (e.g. in Lifshitz theory)
- Geometry
  - It is not all about modes confinement!

Challenges: Geometry

- It is not all about modes confinement:

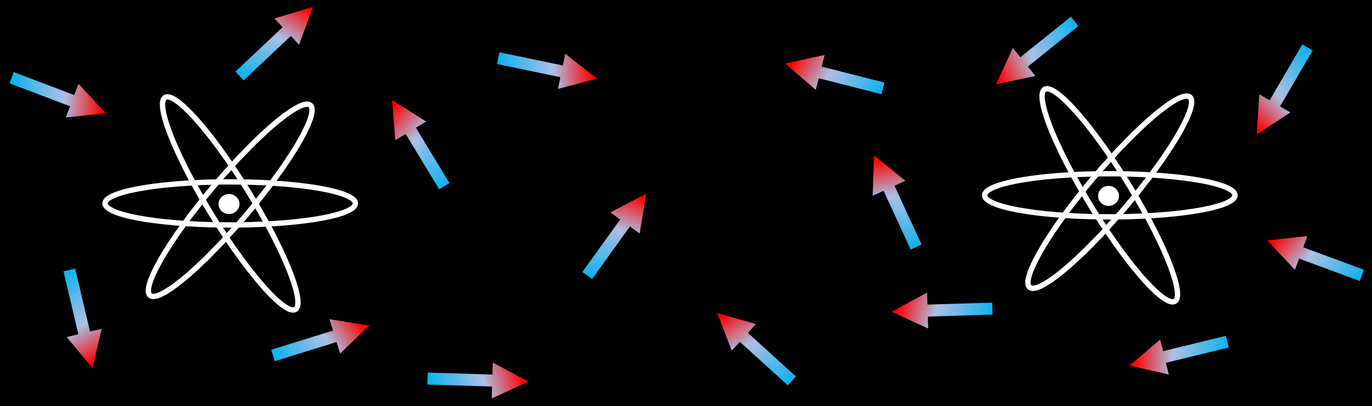


- It's also about the distribution of stress on boundaries!

Questions

- Boundary Conditions relevant in condensed matter and solids?

- Macroscopic to microscopic for fermionic fields?
  - Atoms borrow polarization from fermionic field?
  - Atoms response properties to fermionic fluctuations?



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